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DETERMINATION OF THE SUPERCONDUCTING SKIN DEPTH FROM THE ENERGY GAP AND SUM RULE*

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It has previously been pointed out[1] that the oscillator-strength sum rule provides a useful relation between the energy gap in a superconductor and the strength of the supercurrent which flows in response to an applied low-frequency field. A direct consequence is the effect of impurities in increasing the penetration depth.[2] The purpose of the present note is to extend these arguments to the case of an ideal pure bulk superconductor. In this way we are led to a nonlocal relation of current to field which is independent of any detailed model. This relation is equivalent to that proposed by Pippard[3] on a relatively heuristic basis to explain his experimental results, and by Bardeen, Cooper, and Schrieffer[4] (BCS) on the basis of their detailed microscopic model of superconductivity. Our result is an explicit confirmation of a suggestion made a few years ago by Bardeen,[5] that the electromagnetic properties of a superconductor are immediate consequences of its energy gap. Our proof is, however, more straightforward than Bardeen's, in that it makes use of the Kramers-Kronig relations and the sum rule to introduce the effect of the energy gap. Thus, only matrix elements for true physically observable absorptive processes are involved, while Bardeen's calculation dealt with matrix elements for virtual excitations concerning which it was necessary to make conjectures. The present approach is restricted to oscillating fields and consequently does not actually describe the Meissner effect (expulsion of a dc magnetic field). But the expulsion of an ac field is accounted for, and since the penetration depth which we determine is frequency independent, it follows that it must be equal to that for the dc limit of zero frequency. (See also our concluding remarks.)

To provide a general framework for the analysis, we Fourier-analyze all space- and time-dependent quantities. The coefficients of the plane wave $\exp[i(\vec{q}\cdot\vec{x} - \omega t)]$ in the current density and electric field will be denoted by $\vec{J}(\omega, \vec{q})$ and $\vec{E}(\omega, \vec{q})$, respectively. The ratio of current to field defines a complex conductivity, $\sigma(\omega, q) = \sigma_1(\omega, q) + i\sigma_2(\omega, q)$, which is a function of the wave number q as well as the angular frequency ω. What we seek to establish for the superconducting state is a certain dependence of $\sigma_2(\omega, q)$ on q, for small values of ω. If $\sigma_2(\omega, q)$ were independent of q, then the inverse Fourier transform would yield a delta function in configuration space, corresponding to a strictly local relationship between current density and applied field. On the other hand, if $\sigma_2(\omega, q)$ were to diminish as q increased, then the supercurrent which flowed in response to a spatially varying field would not have its full London value, and the relationship of current to field would be of the same type as the nonlocal one expressed by Pippard's theory. We shall see by arguments based upon quite general principles that the latter situation is the

case.

The concept of a wave number as well as frequency-dependent conductivity is equivalent to a similar concept for the dielectric constant which has been explored thoroughly by Lindhard[6] for the normal state of the metal. Assuming that q is much less than the Fermi wave vector, and making a sign correction in his Eqs. (3.15) and (3.17), we obtain $\sigma_1 = 0$ for $\omega > v_0 q$, and

$$\sigma_1(\omega, q) = \frac{3\pi}{4} \frac{ne^2}{mv_0 q} \left(1 - \frac{\omega^2}{v_0^2 q^2}\right), \tag{1}$$

for $\omega < v_0 q$, where n, e, and m are the density, charge, and mass of the free electrons. An alternative derivation of (1) is to take the limit as $l \to \infty$ of the Fourier transform of the integral expression given by Chambers[7] for the retarded nonlocal relationship of current density to electric field. Explicit integration of (1) yields

$$\int_0^\infty \sigma_1(\omega, q)\, d\omega = \pi n e^2/2m, \tag{2}$$

independent of q. The sum rule[8] states that (2) must hold regardless of the details of the system. (We neglect throughout the small contribution to the conductivity from the motion of the ions.)

If now, for some reason the conductivity is modified so that the area under the $\sigma_1(\omega, q)$ vs ω curve is apparently diminished by the amount A, the sum rule (2) requires that the "lost" area be compensated by a delta function of strength A at $\omega = 0$. This in turn produces a contribution to the imaginary part of the conductivity, according to the Kramers-Kronig causality relations, of amount

$$\sigma_2^A(\omega, q) = 2A/\pi\omega. \tag{3}$$

It is known empirically[9] that the effect of the transition to the superconducting state is to remove an area A which may conveniently be expressed as $\tilde{\omega}_g \sigma_1(0, q)$, the product of an effective gap frequency $\tilde{\omega}_g$ and the value of $\sigma_1(\omega, q)$ at zero frequency. This is true at least when $\sigma_1(\omega, q)$ is nearly constant[10] for $\omega \lesssim \tilde{\omega}_g$, as it is for $v_0 q \gg \tilde{\omega}_g$. The effective gap width $\hbar\tilde{\omega}_g$ defined in this way is about twice the threshold energy $\hbar\omega_g$ or about $8kT_c$. Replacing A in (3) by $\tilde{\omega}_g \sigma_1(0, q)$ and using (1), we find an imaginary conductivity equal to $(ne^2/m\omega)F(q)$, where $F(q)$ measures the q-dependent reduction of the London conductivity given by the first factor. For $q \gg \tilde{\omega}_g/v_0$, we find explicitly

$$F(q) = 3\tilde{\omega}_g/2v_0 q. \tag{4}$$

For $q = 0$, we expect $F(0) = 1$, since for $q \approx 0$, the entire area under $\sigma_1(\omega, q)$ as given by (1) falls well below ω_g, yielding the full sum rule value for A, and hence the full London conductivity.[11] The difference in the two cases is illustrated in Fig. 1.

The penetration depth can now be calculated from the familiar formula[4] for diffuse reflection:

$$\lambda = \pi \Big/ \int_0^\infty \ln[1 + F(q)/q^2\lambda_L^2]\, dq, \tag{5}$$

where λ_L is the London penetration depth $(mc^2/4\pi e^2)^{1/2}$. Assuming (4) to hold for all q of importance in the integration leads to

$$\lambda = \left(\frac{\sqrt{3}\, v_0}{4\tilde{\omega}_g} \lambda_L^2\right)^{1/3}. \tag{6}$$

(Specular surface reflection reduces this by a factor of 8/9.) Pippard's expression[3] in the same limit (λ_∞) is

$$\lambda = \left(\frac{\sqrt{3}}{2\pi} \xi_0 \lambda_L^2\right)^{1/3}, \tag{7}$$

where ξ_0 is his coherence length. Equating (6)

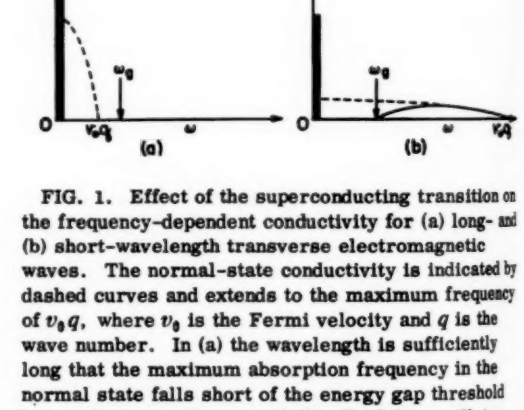

FIG. 1. Effect of the superconducting transition on the frequency-dependent conductivity for (a) long- and (b) short-wavelength transverse electromagnetic waves. The normal-state conductivity is indicated by dashed curves and extends to the maximum frequency of $v_0 q$, where v_0 is the Fermi velocity and q is the wave number. In (a) the wavelength is sufficiently long that the maximum absorption frequency in the normal state falls short of the energy gap threshold $\hbar\omega_g$. Consequently essentially all of the oscillator strength is absorbed by the delta function at zero frequency, leading to a full London current. In (b) the shorter wavelength causes the absorptions in the normal state to be spread over a frequency interval much larger than the energy gap. The strength of the delta function is therefore less and the London current is weakened. This dependence of the London current on wavelength is equivalent to the nonlocal current-field relation of Pippard.

and (7), we find

$$\xi_0 = \frac{\pi}{2}\frac{v_0}{\tilde{\omega}_g} = \left(\frac{\pi k T_C}{2\hbar\tilde{\omega}_g}\right)\left(\frac{\hbar v_0}{k T_C}\right) = a\frac{\hbar v_0}{k T_C}, \qquad (8)$$

where Pippard's constant a is given in our picture by $(\pi/2)(kT_C/\hbar\tilde{\omega}_g) \approx 0.2$ using the empirical gap data mentioned above. For comparison, Faber and Pippard[12] experimentally found $a \approx 0.15$, and BCS gives 0.18. Since λ depends on ξ_0 only through a cube root, this agreement is extremely good.

Summing up, we have obtained Pippard's result for the increase in penetration depth above the London value:

$$(\lambda/\lambda_L) = 0.65(\xi_0/\lambda_L)^{1/3}, \qquad (7')$$

with ξ_0 defined by (8). This result holds only if $\xi_0 \gg \lambda_L$, which is fairly well satisfied in most cases. It is easily shown that the fractional correction in λ resulting from the deviation of $F(q)$ from (4) for small q is positive and of order $(\lambda_L/\xi_0)^{2/3}$ [or $(\lambda_L/\xi_0)^{4/3}$ for specular reflection]. The essential point is that the low-frequency electromagnetic properties of a superconductor are immediate consequences of the application of general principles to empirical infrared absorption data. Thus, the task of a microscopic theory may be properly regarded as being the deduction of the experimentally observed energy gap in the absorption spectrum. If successful in this, it automatically yields the low-frequency supercurrent behavior, including the Meissner effect, although the deduction of the Meissner effect itself follows somewhat different lines and will be contained in a subsequent publication.

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[1] R. A. Ferrell and R. E. Glover, III, Phys. Rev. 109, 1398 (1958).

[2] M. Tinkham, Physica 24, (Suppl.) 35 (1958).

[3] A. B. Pippard, Proc. Roy. Soc. (London) A216, 547 (1953).

[4] Bardeen, Cooper, and Schrieffer, Phys. Rev. 108, 1175 (1957).

[5] J. Bardeen, Phys. Rev. 97, 1724 (1955).

[6] J. Lindhard, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 28, No. 8 (1954).

[7] See Sec. 3 of A. B. Pippard, in Advances in Electronics and Electron Physics, edited by L. Marton (Academic Press, Inc., New York, 1954), Vol. VI.

[8] Although the sum rule was considered in reference 1 to be a consequence of imposing the equality $\lim_{\omega\to\infty}\omega\sigma_2 = ne^2/m$, it can also be derived explicitly from quantum mechanics by a well-known technique which involves using closure and evaluating a double commutator in two different ways. See, for example, R. Kubo, J. Phys. Soc. Japan 12, 570 (1957); and L. Landau and E. Lifshitz, Quantum Mechanics (Addison-Wesley, Reading, 1958), p. 455.

[9] R. E. Glover, III, and M. Tinkham, Phys. Rev. 108, 243 (1957); 110, 778 (1958). P. L. Richards and M. Tinkham, Phys. Rev. Lett. 1, 318 (1958); M. A. Biondi and M. P. Garfunkel, Phys. Rev. Lett. 2, 143 (1959).

[10] For this case we make the assumption that the ratio of $\sigma_1(\omega, q)$ in the superconducting state to its value in the normal state is a universal function of ω/ω_g, regardless of whether the sample under study is a thin film, or an impure or pure bulk specimen.

[11] This conclusion does not follow strictly from general principles, since it is conceivable that some of the oscillator strength could be left above the energy gap, even upon passing to the limit of long wavelengths. In this case the delta function and, in turn, the London current would have only a fraction of the full strength permitted by the sum rule. An ideal pure metal in its normal state does, however, exhibit the full London current for oscillating uniform fields, and it does not seem reasonable to suppose that the superconducting transition would weaken this current. A recent calculation of Wentzel, using Bogoliubov's formulation of the BCS theory [G. Wentzel, Phys. Rev. 111, 1488 (1958)], predicts precisely this behavior, which raises some doubts concerning the validity of the theoretical basis of the calculation. The model of M. R. Schafroth and J. M. Blatt [Phys. Rev. 100, 1221 (1955)] also suffers from this defect.

[12] T. E. Faber and A. B. Pippard, Proc. Roy. Soc. (London) A231, 336 (1955).

DEFORMATION POTENTIAL IN GERMANIUM FROM OPTICAL ABSORPTION LINES FOR EXCITON FORMATION

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In their study of the direct exciton in germanium, Zwerdling et al.[1] observed a second absorption peak (1.5°K) about 0.005 ev above the absorption identified with the direct exciton. Since the separation of the two peaks was several times greater than the binding energy of the exciton, this could not be an excited state of the exciton in the ordinary sense, and no explanation has heretofore been given. However, recent experiments by Macfarlane et al.[2] have established that the ZLRB sample was strained, due to the difference in thermal contraction of the sample and its glass substrate, and that the strain accounted for the observed discrepancy in the position of the exciton line reported by the two groups, and for the presence of the second peak in the ZLRB data.

The appearance of the second peak should actually be expected in the presence of shear strain, since shear splits the valence band edge into two edges. There will then be two exciton levels, one associated with each valence band edge, as indicated in Fig. 1. The splitting and shift of the exciton with strain can, in fact, be used to obtain information about the deformation potential for the band edges. To analyze the data here we shall assume that the exciton binding energies ϵ_+ and ϵ_- are independent of strain, that is, equal to the binding energy in the absence of strain. This is reasonable since the binding energy is determined largely by the small electron mass. The splitting 2Δ of the valence band edge and the mean shift $\bar{E}-E_0$ of the gap are then determined from the position of the two exciton lines in the presence of strain[1] and the single exciton line in the absence of strain.[3]

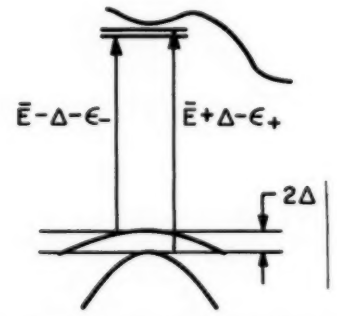

FIG. 1. An illustration of direct exciton formation transitions in Ge with general homogeneous strain present. (The exciton levels have been placed below the conduction band edge by an amount equal to their binding energies, and are not to be confused with one-electron levels.) 2Δ denotes the splitting of the valence band, $\bar{E}$ the mean gap between the valence bands and the conduction band ($\bar{E}=E_0$ in the absence of strain), and ϵ_+ and ϵ_- the binding energies of the two excitons.

The effect of a homogeneous strain S_i ($i=1, 2, \ldots, 6$, where $S_4=2S_{yz}$) on the conduction and valence band edges is conveniently described by Hamiltonians derived by symmetry considerations

$$\mathcal{H}^c=\mathcal{H}_0{}^c+D_d{}^c(S_1+S_2+S_3), \tag{1}$$

$$\mathcal{H}^v=\mathcal{H}_0{}^v+D_d{}^v(S_1+S_2+S_3)+\tfrac{2}{3}D_u[(J_x{}^2-\tfrac{1}{3}\vec{J}^2)S_1+\text{c.p.}]$$
$$+\tfrac{2}{3}D_u'[\tfrac{1}{2}(J_yJ_z+J_zJ_y)S_4+\text{c.p.}], \tag{2}$$

where the D's are deformation potential coefficients, $\hbar\vec{J}$ is the angular momentum of the hole ($J=3/2$), and the $\mathcal{H}_0$'s describe the situation in the absence of strain; D_d is the shift per unit dilatation of the band edge, while $|2D_u|$ is the splitting of the band edge induced by uniaxial shear strain per unit extension along the [001] axis; $|2D_u'|$ gives the splitting similarly for the [111] axis.[4] Finally, "c.p." means "cyclic permutations of x, y, and z," referring to the crystallographic axes. The deformation potential parameters $D_d{}^c-D_d{}^v$, $|D_u|$, and $|D_u'|$ may be determined from the experimental splitting and mean shift measured for appropriate strains.

The data considered here[1] are for a Ge sample in the form of a very thin slab of uniform thickness with slab normal in the [110] direction. The sample is cemented to a supporting substrate at roughly room temperature. When the temperature is changed, the difference between the thermal expansion of Ge and the substrate induces a strain in the sample. If the sample, substrate, and cementing are homogeneous and the substrate is elastically isotropic parallel to the interface, as assumed here, then the symmetry implies that the strain tensor in the sample is uniform,

Table I. Splitting of the valence band and mean shift of the direct energy gap for strains induced in thin slab samples, for three directions of the slab normal.

Slab normal	[001]	[111]	[110]
Strain components	$S_1=S_2=S,\ S_3=-\lambda S,$ $S_4=S_5=S_6=0$	$S_1=S_2=S_3=\frac{1}{3}(2-\lambda)S,$ $S_4=S_5=S_6=-\frac{1}{3}(1+\lambda)S$	$S_1=S_2=\frac{1}{2}(1-\lambda)S,\ S_3=S,$ $S_4=S_5=0,\ S_6=-(1+\lambda)S$
λ	$2c_{12}/c_{11}$	$2(c_{11}+2c_{12}-2c_{44})/(c_{11}+2c_{12}+4c_{44})$	$(c_{11}+3c_{12}-2c_{44})/(c_{11}+c_{12}+2c_{44})$
Δ	$\frac{2}{3}(1+\lambda)\lvert D_u\rvert\lvert S\rvert$	$\frac{2}{3}(1+\lambda)\lvert D_u'\rvert\lvert S\rvert$	$\frac{2}{3}(1+\lambda)(\frac{1}{2}D_u^{\,2}+\frac{3}{4}D_u'^2)^{1/2}\lvert S\rvert$
$\bar{E}-E_0$	$(2-\lambda)(D_d^{\,c}-D_d^{\,v})S$	$(2-\lambda)(D_d^{\,c}-D_d^{\,v})S$	$(2-\lambda)(D_d^{\,c}-D_d^{\,v})S$
R	$\lvert D_u\rvert/\lvert D_d^{\,c}-D_d^{\,v}\rvert$	$\lvert D_u'\rvert/\lvert D_d^{\,c}-D_d^{\,v}\rvert$	$(\frac{1}{4}D_u^{\,2}+\frac{3}{4}D_u'^2)^{1/2}/\lvert D_d^{\,c}-D_d^{\,v}\rvert$

diagonal with respect to coordinate axes in the plane of the slab (x', y') and perpendicular to the slab (z'), and isotropic in the plane of the slab. We denote the principal strain components by S, S, $-\lambda S$, where λ is determined by the condition that the normal stress vanish. When referred to crystallographic coordinates, the strain components are $S_1=S_2=\frac{1}{2}(1-\lambda)S$, $S_3=S$, $S_4=S_5=0$, $S_6=-(1+\lambda)S$. With these values of the strain, Δ and $\bar{E}-E_0$ are readily determined from the eigenvalues of (1) and (2). The results are given in Table I together with corresponding results for the slab normal in the [001] and [111] directions. The sign of $D_d^{\,c}-D_d^{\,v}$ is determined by the sign of S. If the magnitude of S is unknown, valuable information can still be obtained in the form of the ratio,

$$R=\frac{3(2-\lambda)}{2(1+\lambda)}\,\frac{\Delta}{|\bar{E}-E_0|}, \qquad (3)$$

of shear to dilatation deformation potential coefficients (See Table I).

Values of R derived from the ZLRB and the MMQR[3] data are given in Table II and are probably in error by less than 10%.[5] The close agreement between the values of R at the two temperatures supports our interpretation of the data.

If we use the value -9 ev per unit dilatation for $D_d^{\,c}-D_d^{\,v}$ obtained from pressure measurements,[6] we can deduce from R the value 4.5 ev per unit strain for $(D_u^{\,2}/4+3D_u'^2/4)^{1/2}$. This is somewhat larger than the result $E_2=1.66$ ev per unit strain obtained by Brooks,[4] from piezoresistance data, where $-E_2$ is an average of D_u and D_u'. The agreement is slightly improved if we use Brooks' formula (6.34) to calculate D_u and D_u' separately from the piezoresistance coefficients $(m_{11}-m_{12})/2$ and m_{44}, respectively, giving $D_u=0.3$ ev and $D_u'=-2.9$ ev, which yields $(D_u^{\,2}/4+3D_u'^2/4)^{1/2}=2.5$ ev. This may not be significant, however, as the piezoresistance calculation neglects anisotropy of the energy surfaces.

Further strain measurements on samples with [001] and [111] slab normals would give D_u and D_u' separately, and in addition it should be possible to determine the signs of D_u and D_u' by analyzing the magnetic spectrum. Finally, measurements should be made on substrates with known thermal expansion (so that S can be accurately determined).

Table II. The ratio R of shear to dilatation deformation potential coefficients derived from splitting and mean shift of exciton lines.[a]

T(°K)	λ	Splitting 2Δ (milli-ev)	Mean shift $\lvert\bar{E}-E_0\rvert$ (milli-ev)	R
1.5	0.442	4.9	8.9	0.46
77	0.442	4.1	6.9	0.48

[a]Elastic constant data were taken from M. E. Fine, J. Appl. Phys. 26, 862 (1955).

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[1]Zwerdling, Lax, Roth, and Button, Phys. Rev. (to be published).

[2]Macfarlane, McLean, Quarrington, and Roberts, Phys. Rev. Lett. 2, 252 (1959).

[3]Macfarlane, McLean, Quarrington, and Roberts, Proc. Phys. Soc. (London) 71, 863 (1958).

[4]H. Brooks, Advances in Electronics and Electron Physics (Academic Press, New York, 1955), Vol. 7, p. 85. Brooks uses $E_1=-D_d^{\,v}$, $E_2=-D_u=-D_u'$ in

Eq. (6.29).

[5]Since the magnitude of S is unknown for the ZLRB data, only values of R can be derived. The sign of S is known,[2] however, implying $D_d^c - D_d^v < 0$ as expected.

[6]W. Paul and D. M. Warschauer, J. Phys. Chem. Solids 5, 89 (1958).

ELECTRICAL AND THERMAL RESISTIVITY OF DISLOCATIONS

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For a long time it has been known that mechanical deformation greatly reduces the electrical conductivity of metals.[1-3] Since the change in resistivity was consistently found to be one to two orders of magnitude greater than expected theoretically[4-8] for scattering by the strain field of a dislocation, various authors have invoked scattering by stacking faults,[9] or the dislocation core,[10] to explain the discrepancy. Because of the difficulty of the latter calculations, the reliability of such explanations has not been determined.

Recently, observations of the effect of dislocations on the thermal conductivity of LiF at low temperatures by Sproull, Moss, and Weinstock[11] have shown that the thermal conductivity is about 100 times less than that predicted by Klemens.[12] It seems that this discrepancy may be intimately related to that of the electrical resistivity caused by dislocations. The author[13] has proposed an explanation of the high thermal resistivity caused by dislocations. The purpose of the present note is to suggest that the scattering of electrons by the strain field of an edge dislocation may be much larger than found in previous calculations.[4-8] Also, recent experimental results are compared to the author's theory[13] of thermal resistance of dislocations.

The raison d'être of the electron problem may be best understood by comparing the author's theory of the strain field scattering of phonons with that of Klemens.[12] In both treatments the scattering proceeds by the anharmonic forces. The energy-conserving perturbation is then linear in the displacement field and quadratic in the phonon creation and annihilation operators. The author's theory is atomic in character; the complicated sums are rearranged and simplified without approximation. The approximation necessary to work out a given problem are made after the pertinent physical quantities have been separated out. The essential feature is that the Fourier component of the strain field displacement enters into the matrix elements.

Klemens,[12] proceeding from a Grüneisen model of the anharmonicities, obtains in the pertinent matrix elements the Fourier component of the dilatation (or rotation). In deriving this result (for a simple cubic lattice), the approximation was made that the component of the difference of displacements $\vec{v}$ at lattice sites $\vec{m}$ and $\vec{m}-\vec{l}$ in the direction of the linkage l_i is equal to $(dv_i/dx^i)a$, where a is a lattice constant. The details of the calculation are such that the (two-dimensional) Fourier component of this function is then taken. (That is, the approximation is made before the sum over all lattice positions is taken.) It is easy, using the exact difference of y-displacements due to an edge dislocation, to demonstrate that the integrand is not uniformly convergent in Klemens' approximation. Thus the author's calculation for the relaxation time τ of a phonon in the strain field of an edge dislocation differs from Klemens' result essentially by the factor $\frac{1}{2}[\ln(R/r_0)]^2$. This result is

$$\tau^{-1} = \tfrac{1}{2}\sigma[\ln(R/r_0)]^2 b^2\gamma^2 c\, q. \qquad (1)$$

Here σ is the dislocation density, r_0 is the core radius, b is the Burgers vector, R is the mean range of the dislocation strain field, γ is Grüneisen's constant, c is the velocity of sound, and q is the wave vector of the phonon. The logarithmic factor directly reflects the long-range (logarithmic) nature of the displacement field, i.e., the scattering "center" is not localized.

We now return to the problem of electron scattering by the strain field of a dislocation. In this case it is not evident that the core scattering is negligible, since the wavelengths of the electrons in conductivity are much smaller than those of the phonons considered above. Further complications arise from the redistribution of the electrons in a dilated lattice.[4] Harrison[10] has shown that for a hollow-core dislocation model, the scattering is almost enough to account for the experimental results. However, it seems that such

a sharp discontinuity would overestimate the scattering. The author does not wish to discount the contribution of core scattering, but merely wishes to point out that the strain field scattering may be large enough to account for the reduction of conductivity by dislocations, contrary to previous results.[4-8]

According to the present view, calculations of the scattering of a pair of edge dislocations of opposite sign[5-7] no more represents the true scattering than, say, the scattering of an electric dipole approximation represents Rutherford scattering, since the logarithmically varying displacements will cancel at large distances for such a pair. The only other calculations known to the author are based on either strain components[4] or dilatation (or rotation).[8] It is seen that the latter calculations are open to the same objections as were raised above in regard to Klemens' calculation of phonon scattering. However, Hunter and Nabarro[4] raise objections to using a perturbation containing the displacement, chiefly because for a dislocation the displacement is large, so that the perturbation is not necessarily small. For example, in the rigid-ion model the perturbation H' is

$$H' = -\sum_{\vec{m}} \vec{v}_{\vec{m}} \cdot \nabla U(\vec{r}-\vec{m}), \qquad (2)$$

where $\vec{v}_{\vec{m}}$ is the strain field displacement at position $\vec{m}$, $\vec{r}$ is the position of the electron, and U is the (periodic) potential seen by the electron. Nevertheless, because of the previous remarks we proceed to investigate Eq. (2). From a practical point of view, this is no disadvantage.[14] It is then surmised that a more accurate treatment would not change the result in any essential way even if the range were greater than that for which the perturbation method is valid.

We note that Eq. (2) is linear in $\vec{v}_{\vec{m}}$, as was the case for the scattering of phonons described previously. It is now most convenient to compare with Klemens' treatment[8] of electron scattering by dislocations, since he casts the theory into a form equivalent to his calculation of phonon scattering.[12] (His result agrees with that of Hunter and Nabarro to within a factor of 3.)

Thus it is proposed that a better value for the resistivity is found by multiplying previous values by the factor[15] $\frac{1}{2}[\ln(R/r_0)]^2$ found previously[13] for the case of thermal conductivity. The value of R may be estimated as follows: randomly oriented dislocations, $R \approx \sigma^{-1/2}$; for n dislocations lying in the same slip plane, $R \approx n\sigma^{-1/2}$. The values of n for which this correction produces agreement with experiment are listed in Table I. The experimental values of σ are deduced from stored energy or x-ray measurements. σ_K is the density of dislocations required to account for the resistivity as computed from Klemens' formula.[8] The order of magnitude of n is consistent with our knowledge of the distribution of dislocations in deformed crystals. Even if n were as small as unity, the order of magnitude of the scattering would be the same as that observed.

We now present experimental evidence for the validity of Eq. (1) for thermal conductivity. In Table II are listed the experimental values of thermal conductivity of LiF at 4°K obtained by Sproull, Moss, and Weinstock,[11,16] and the value of n required to obtain agreement with the conductivity calculated from Eq. (1). The quoted densities were found by etch-pit counting as described by Gilman and Johnston.[17] It will be noted that the reduction of conductivity caused by quenching is considerably less than in the compressed specimens, even though the densities

Table I. Density of dislocations as deduced from electrical resistivity, σ_K[a]; stored energy and x-ray measurements σ(exp). n is the number of similarly oriented dislocations required to give agreement of σ(exp) with the present theory.

Substance	σ(exp), cm^{-2}	σ_K, cm^{-2}	n
Ni[b]	2.7×10^{11}	1.5×10^{13}	10^4
Cu[c]	8.5×10^{10}	1.7×10^{12}	10^2
Brass[d]	3×10^{12}	6×10^{13}	10^3

[a]See reference 8.
[b]See reference 1.
[c]See reference 2.
[d]See reference 3.

Table II. Thermal conductivity[a,b] Q of LiF at 4°K for quenched and compressed specimens. n is computed using Eq. (1). The annealed specimen had $Q=6$ watts/cm deg at this temperature and $\sigma=3\times10^4$ cm^{-2}.

Treatment	Quench	2% reduction	4% reduction
Q (watts/deg cm)	2.3	0.7	0.25
σ(cm^{-2})	3×10^7	1.8×10^7	4.6×10^7
n	1	10^3	10^4

[a]See reference 11.
[b]See reference 16.

are comparable. This fact, and the assignment of n in Table II, are understandable from the work of Gilman and Johnston on dislocations in LiF, in which it was shown that dislocations introduced by quenching were in the form of individual loops, while mechanical deformation produced slip bands of thousands of dislocations. It should also be pointed out that the increase of scattering due to the alignment of dislocations in slip planes is of quite a different nature than that proposed by Klemens,[12] since the effective phonon wavelength is considerably less than the mean interdislocation distance $\sigma^{-\frac{1}{2}}$.

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*National Science Foundation Predoctoral Fellow.

[1]W. Boas, Dislocations and Mechanical Properties of Crystals (John Wiley and Sons, Inc., New York, 1957), p. 333, and references.

[2]Clarebrough, Hargreaves, and West, Acta Met. 5, 738 (1957); Phil. Mag. 1, 528 (1956).

[3]Kemp, Klemens, Tainsh, and White, Acta Met. 5, 303 (1957).

[4]S. C. Hunter and F. R. N. Nabarro, Proc. Roy. Soc. (London) A220, 542 (1953).

[5]J. S. Koehler, Phys. Rev. 75, 106 (1949).

[6]J. K. Mackenzie and E. H. Sondheimer, Phys. Rev. 77, 264 (1949).

[7]D. L. Dexter, Phys. Rev. 86, 770 (1952).

[8]P. G. Klemens, Can. J. Phys. 34, 1212 (1956).

[9]A. Seeger, Can. J. Phys. 34, 1219 (1956).

[10]W. Harrison, J. Phys. Chem. Solids 5, 44 (1957).

[11]Sproull, Moss, and Weinstock (to be published).

[12]P. G. Klemens, Proc. Phys. Soc. (London) A68, 1113 (1955); Solid State Physics (McGraw-Hill Book Company, New York, 1958), Vol. 7, p. 1.

[13]P. A. Carruthers, Phys. Rev. (to be published).

[14]$V_y < b$ for $R/r_0 < 1000$. For metals with $\sigma \sim 10^{11}$ cm^{-2}, $R \sim \sigma^{-\frac{1}{2}} \sim 50 r_0$.

[15]The exactness of the factor 1/2 is not to be taken seriously.

[16]Sproull, Moss, and Weinstock (private communication).

[17]J. J. Gilman and W. G. Johnston, Dislocations and Mechanical Properties of Crystals (John Wiley and Sons, Inc., New York, 1957), p. 116.

De HAAS–van ALPHEN TYPE OSCILLATIONS IN THE INFRARED TRANSMISSION OF BISMUTH

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(Received March 30, 1959)

We have found that single crystals of pure bismuth show a pass band for radiation between approximately 20 and 50 microns. A transmission spectrum for a crystal 4×10^{-3} cm thick at 2°K is shown in Fig. 1. The end of the pass band at long wavelength in the region of 55 microns is associated with the plasma frequencies and has been discussed previously in relation to reflection type experiments.[1] In this note we shall discuss only the measurements that have been made in the short-wavelength region of the pass band. The most striking feature that we observe here is a magneto-oscillatory transmission coefficient which is periodic in $1/H$. This property is shown in Fig. 2 which is a reproduction of the recorder trace of the transmitted signal through a crystal. The direction of propagation and the

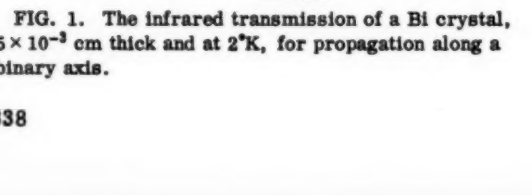

FIG. 1. The infrared transmission of a Bi crystal, 5×10^{-3} cm thick and at 2°K, for propagation along a binary axis.

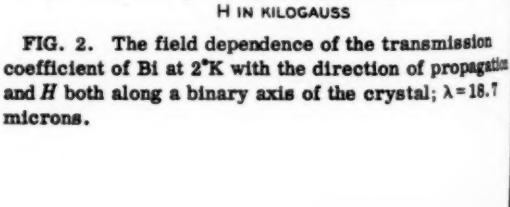

FIG. 2. The field dependence of the transmission coefficient of Bi at 2°K with the direction of propagation and H both along a binary axis of the crystal; $\lambda = 18.7$ microns.

magnetic field were both along a binary axis, the wavelength of the radiation was fixed at 18.7 microns, and the magnetic field increases uniformly from left to right. Measurements made at a fixed magnetic field show no such oscillations as a function of frequency, but rather changes in both the slope and position of the end of the pass band in such a way as to be consistent with the results shown in Fig. 2 where the magnetic field is the variable.

The experimental conditions necessary to observe the results shown in Fig. 2, though readily reproduced, are critically dependent on both the orientation and perfection of the sample. A misorientation of 3° splits the pattern into two separate sets of oscillations which are difficult to resolve at low fields. The doublet structure at small fields which appears in Fig. 2 is due, in fact, to just such a small misorientation, in this case, of less than 0.5°. We have also found that unless the samples were heavily etched to remove strain, all of the oscillatory effects were absent.

The period (in $1/H$) of the oscillations shown in Fig. 2 is such that if we ascribe this effect to the low-mass electron valleys in the conduction band, use a cyclotron mass of $0.01m_0$ and an expression $E_f = (n+\gamma)\hbar\omega_c$ for the positions of the maxima or minima, we obtain a Fermi energy E_f of 0.017 ev. Here γ is a constant phase factor and ω_c is the cyclotron angular frequency eH/m^*c.

The cause of the magneto-oscillations in transmission and the reason for the high-frequency cutoff are elucidated to some extent by experiments performed on samples which have been doped with small quantities of a tin impurity. It has been established previously that this lowers the Fermi energy and so reduces the number of electrons and increases the number of holes. We find that in these tin-alloyed samples, the edge at 20 microns moves to longer wavelengths and the period in $(1/H)$ of the oscillations increases. Using the same value for the cyclotron mass for the low mass of the electrons, we find that the change in the Fermi energy is equal to the shift in energy of the edge of the pass band.

These data can be interpreted in part according to the following model. The zero-field end of the pass band occurs when it becomes energetically possible to excite electrons from a lower lying band or group of discrete states up to the Fermi surface. It must be a transition up to the Fermi surface, rather than from the Fermi surface up to a higher lying state, because of the motion of the edge to longer wavelengths when the Fermi energy is lowered. The position of the edge at 20 microns would set the lower lying states about 0.05 ev below the Fermi surface. The oscillatory absorption then follows as a consequence of the oscillations in the Fermi level produced by the usual magnetic field dependence of the density of states in the low-mass electron band. Brailsford[2] has considered the problem of the motion of the Fermi level with magnetic field in a semimetal like bismuth with overlapping bands. The shape of the oscillations in the Fermi level predicted by this theory and the $1/H$ periodicity agree with the experimental curve shown in Fig. 2. This mechanism does not, however, provide an explanation for the monotonic decrease in transmission with field.

The authors are pleased to acknowledge the help obtained in discussion of the interpretation of these results with A. D. Brailsford and L. C. Hebel.

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[1]Boyle, Brailsford, and Galt, Phys. Rev. 109, 1396 (1958).

[2]A. D. Brailsford (private communication and to be published).

OSCILLATORY MAGNETO-ACOUSTIC EFFECT IN METALS

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(Received March 30, 1959)

This note reports the results of a calculation of ultrasonic absorption in metals in the presence of a magnetic field transverse to the direction of propagation. Two cases have been investigated in detail: (a) longitudinal wave, and (b) shear wave with polarization, $\vec{u}$, perpendicular to field, $\vec{H}$ (as well as to the wave-vector, $\vec{q}$). The method of treatment involves the combined application of the Boltzmann transport equation and Maxwell's equations to the free-electron model,[1] and is thus essentially the same as that employed by Rodriguez.[2] However, the present results differ considerably from those of R; in particular, as shown by the accompanying figures, substantial magneto-acoustic oscillations of the general type predicted by Pippard,[3] and observed experimentally,[4] are obtained.

The results are shown in Figs. 1 and 2. These may be regarded as the attenuations plotted versus (ω/H) for a given sample (fixed τ), for a series of different frequencies (corresponding to different ql). The abscissa is the dimensionless variable $\beta = 2\pi v_0 \omega_c^{-1} = 2\pi D\lambda^{-1}$, where D is the equatorial magnetic diameter. The attenuation coefficients (α) may be obtained by multiplying the numbers shown as ordinates by $(mM^{-1})\tau C_s$.[5]

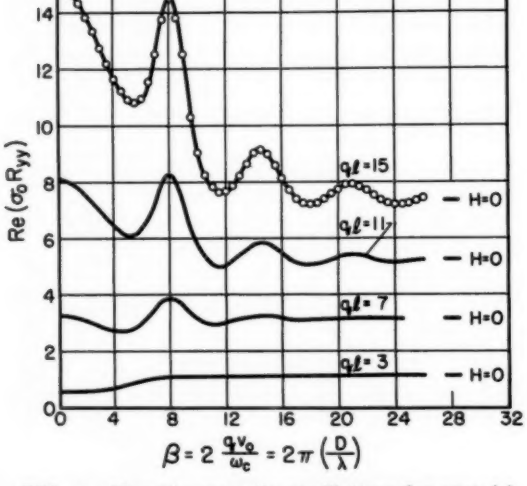

FIG. 1. Magneto-acoustic oscillations for case (a) (longitudinal). All the curves were obtained by computation at the points indicated for $ql = 15$.

The maxima occur at very nearly the same values in both cases. It will be noted that the distances between successive maxima are nearly equal and very close to 2π; the corresponding differences in D are then essentially equal to λ. This confirms the surmise of Pippard[3] that the periodicity is related to the orbit of maximum dimension. The positions of the peaks, however, are not entirely in accord with his predictions. The peak at highest field occurs at $\beta \sim 8.4$, corresponding to the fundamental resonance of an "average" orbit $(\sim 0.75D)$. As the field is decreased, accurate periodicity is gradually attained. It seems reasonable to suppose that this periodicity will obtain for many real Fermi-surfaces in the limit of orbit dimension large compared to λ. It should be pointed out, however, that since the oscillations are appreciable only for[6] $\omega_c\tau \gtrsim 1$, rather large values of ql may be required.

The theory requires the calculation, from the Boltzmann equation, of the three "magneto-acoustic" conductivities σ_{xx}, σ_{xy}, and σ_{yy}, which relate the local electron currents to the electromagnetic "deformation fields" accompanying the sound wave. The specific expressions turn out to be

$$\sigma_{yy} = 3\sigma_0(1 + i\omega\tau)(ql)^{-2} M_\gamma(\beta), \tag{1}$$

$$\sigma_{xy} = \frac{-3}{ql}\sigma_0 \frac{\partial}{\partial\beta} M_\gamma(\beta), \tag{2}$$

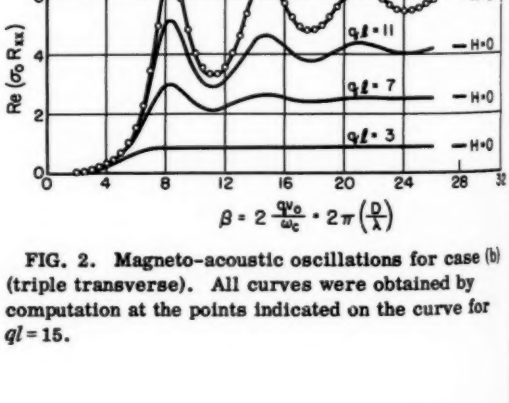

FIG. 2. Magneto-acoustic oscillations for case (b) (triple transverse). All curves were obtained by computation at the points indicated on the curve for $ql = 15$.

$$\sigma_{yy} = \tfrac{3}{2}\sigma_0(1+i\omega\tau)^{-1}\{1-[1+(3+4\gamma^2)\beta^{-2}]M_\gamma(\beta) + 3\beta^{-2}L_\gamma(\beta) - 3\beta^{-1}\frac{\partial}{\partial\beta}L_\gamma(\beta)\}. \quad (3)$$

Here

$$\gamma=(\omega_c\tau)^{-1}(1+i\omega\tau),\quad M_\gamma(\beta)=\frac{1}{\beta}\int_0^\beta L_\gamma(\beta')\,d\beta', \quad (4)$$

$$L_\gamma(\beta)=s_{1,2i\gamma}(\beta)=1-\frac{\pi\gamma}{\sinh 2\pi\gamma}[\bar{J}_{2i\gamma}(\beta)+\bar{J}_{-2i\gamma}(\beta)]. \quad (5)$$

The functions $s_{\mu\nu}(z)$ and $\bar{J}_\nu(z)$ are referred to by Watson[7] as Lommel's function and Anger's function, respectively. They are defined on pages 345 and 308 of reference 7.

In case (a) the present analytic expression for the attenuation coefficient agrees with that given by R's Eqs. (62) and (64). However, the expression obtained in case (b),

$$\alpha=\frac{m}{M\tau C_S}\,\mathrm{Re}\left\{\frac{\sigma_0\sigma_{yy}}{\sigma_{xx}\sigma_{yy}\sigma_{xy}^2}-1\right\}, \quad (6)$$

differs from that given by R's Eq. (54). The latter, in the opinion of the present authors, is incorrect; a basic ingredient of its derivation, namely the assumption of zero Hall field in the propagation direction, can be shown to be inconsistent with Maxwell's equations (in particular with the Poisson relation). In fact, under conditions commonly prevalent in metals (high electron densities and conductivities), Poisson's equation is essentially equivalent to the requirement of quasi-neutrality; in the case at hand, this requirement implies zero Hall-current (rather than field). The resulting difference has an especially marked effect on the high-field behavior; in this domain, as shown by Fig. 2, the attenuation is proportional to H^{-2}, rather than to H^2 as concluded by R.

In order to obtain numerical results, the magneto-acoustic conductivities must be computed from Eq. (1) to (5). The replacement of $\bar{J}_\nu(\beta)$ by its expansion[7] in powers of β yields (absolutely convergent) series for the σ_{ij} which coincide with R's Eqs. (29), (30), and (31). On the other hand, the asymptotic ($\beta \gg 1$, $|\gamma| \ll \beta$) expressions for the σ_{ij} [obtained from the asymptotic series for $\bar{J}_\nu(\beta)$ given by Eq. 10.14 (1) of reference 7], differ from R's Eqs. (33), (34), and (35) in that they contain terms proportional to the Bessel functions $J_{\pm 2i\gamma}(\beta)$, as well as to integrals and derivatives of these functions. It has now to be remarked that the oscillatory behavior of the σ_{ij} (and hence of the attenuation) arises entirely from these additional "Bessel" terms; their omission from R's asymptotic formulas may explain the absence of oscillatory behavior in his final results.

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[1] A. B. Pippard, Phil. Mag. 46, 1104 (1955).

[2] S. Rodriguez, Phys. Rev. 112, 80 (1958), to be referred to as "R".

[3] A. B. Pippard, Phil. Mag. 2, 1147 (1957).

[4] R. W. Morse and J. D. Gavenda, Phys. Rev. Lett. 2, 250 (1959).

[5] v_0 = Fermi velocity; ω_c, ω = cyclotron and sound (angular) frequencies; τ = electron scattering time; m, M = electron and atom masses; λ, C_S = relevant wavelength and velocity of sound.

[6] This requirement on $\omega_c\tau$ may be noted from Figs. 1 and 2 and is directly deducible from asymptotic expansions of the results below.

[7] G. N. Watson, Bessel Functions (Cambridge University Press, Cambridge, 1952, second edition, Chap. 10, especially Secs. 10.1, 10.11, 10.14, 10.15, and 10.7 to 10.75.

ELECTRON PARAMAGNETIC RESONANCE OF MANGANESE IV IN $SrTiO_3$*

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(Received March 30, 1959)

We have investigated the paramagnetic resonance spectrum of a single crystal of $SrTiO_3$ grown by the Verneuil process with the addition of 0.01 weight percent of MnO_2. The measurements were made at room temperature, liquid air temperature (where the $SrTiO_3$ begins to transform from the high-temperature cubic into a tetragonal phase[1]), and at liquid nitrogen temperature. Several samples cut from the crystal were investigated. The magnetic field was varied parallel to the (110) and (100) planes. A spectrometer working at 3.2 cm was used.

Six main hyperfine line groups arising from the nuclear spin $I=\frac{5}{2}$ of the isotope Mn^{55} were ob-

served. The g-factor and the hyperfine constant of the spectrum were found to be

$$g = 1.994 \pm 0.001,$$

$$|A| = 75 \pm 1 \text{ gauss}.$$

The groups showed a fine-structure splitting dependent on the magnetic field direction to the crystalline axis. The splitting was largest for H parallel to the [100] direction. Therefore we conclude that the manganese atoms are situated at lattice points in the $SrTiO_3$. Two positions are possible: at Sr^{2+} sites as Mn^{2+} or Mn^{3+} and at Ti^{4+} sites as Mn^{2+}, Mn^{3+}, or Mn^{IV}.

We do not think that the resonance is due to manganese atoms sitting at a Sr^{2+} place. For Mn^{2+} a g-value as small as 1.994 has not been reported. With hyperfine splitting constants A less than 80 gauss, the experimental g-values are found to be higher than 2.001. As the sign of the estimated crystalline electric field potential is opposite to that of the octahedral Ti^{4+} position, the relaxation time of a $Mn^{3+}(3d^4)$ at a Sr^{2+} place is expected to be so short that no resonance is observable at room temperature. Furthermore, the ionic radius of Mn^{2+} is 0.90 A, that of Mn^{3+} is 0.70 A,[2] while that of Sr^{2+} is 1.27 A which would give a poor fit into the lattice.

The g-value argument also excludes the possibility of a Mn^{2+} at a Ti^{4+} site. In addition, since the Mn^{2+} would be surrounded by six nearest oxygen neighbors as in MgO at a Mg^{2+} position,[3] the hyperfine splitting constants A should nearly coincide as they do for Cr^{3+}.[3,4] This is clearly not the case. The Mn^{3+} is isoelectronic with the Cr^{2+}. Wertz et al.[5] have found no resonance of Cr^{2+} present in MgO. The effective spin of Mn^{3+} in the octahedral field would be 2. With such a spin it seems difficult to explain the sharp lines found in the tetragonal phase at 77°K which are quite independent of the direction of the magnetic field.

These lines can be explained easily by a $\Delta M = \pm\frac{1}{2}$ transition of the only remaining possibility of a four-valent Mn atom ($3d^3$) with spin $S = \frac{3}{2}$ at a Ti^{4+} position. Several reasons support this: The MnO_2 is isomorphous with the TiO_2 which together with SrO was used to grow the crystal. For every Sr^{2+} incorporated at a La^{3+} site in $LaMnO_3$, a Mn^{3+} changes into a Mn^{IV}.[6] The Mn^{IV} is isoelectronic with the Cr^{3+}. Both have an effective S ground state. The resonance of chromium in $SrTiO_3$ arising from Cr^{3+} at a Ti^{4+} lattice point has been detected with a g-value of 1.9788 ± 0.0007.[4] The g-factor of $Mn^{IV}(3d^3)$ is considerably higher; it is nearly the same as that of the $Cr^{III}(3d^3)$ in the $[Cr(CN)_6]^{3+}$ complex[7] where strong covalent bonding is present. Our experiment shows therefore that there is strong covalent bonding between the four-valent manganese and its 6 oxygen neighbors. Therefore we write Mn^{IV} and not Mn^{4+}. Preliminary optical absorption measurements at 300°K indicate a strong absorption line somewhat higher than 24 000 cm^{-1},[8] which we assign to the $\Gamma_2 - \Gamma_5$ transition. With a spin-orbit coupling constant of 133.5 cm^{-1}, one sees immediately that the theory of covalent bonding of Owen[9] concerning charge transfer is no longer valid for this case and overlap of the wave functions must also be taken into account. Such a theory has to our knowledge not been developed for the ($3d^3$) configuration and it would not only be of interest to analyze our data but it might be of help in understanding more quantitatively the phenomena of indirect exchange coupling of -M-O-M-, $M = 3d^3$, in AMO_3 perovskite structure magnetic materials.

Owen[9] has shown that in $[M(H_2O)_6]$ complexes, $M = 3d^n$, the covalent bonding is stronger for M^{3+} than for M^{2+} ions with the same configuration. Presumably the greater positive charge causes an increased tendency of the oxygen electrons to move into the central M-ion bonding molecular orbitals. Combining the resonance result of Cr^{3+} in $SrTiO_3$ with the optical absorption data,[8] one finds medium-strength covalent bonding for the Cr^{3+} using Owen's theory which in this case is applicable. We have herewith a proof that for the ($3d^3$) configuration in the same lattice position of $SrTiO_3$ the four-valent atom has a stronger covalent bonding than the three-valent. This increases the possibility that the greater charge is the cause of stronger bonding and one should perhaps also write $Ti^{IV}(3d^0)$ instead of Ti^{4+} in $ATiO_3$ perovskites.

In Fig. 1 the resonance spectrum is seen for H parallel to a [100] direction together with the DPPH calibration line and the theoretical line positions for a spin Hamiltonian of

$$\mathcal{H} = g\beta\vec{H}\cdot\vec{S} + A\vec{I}\cdot\vec{S} + D\{S_z^2 - \tfrac{1}{3}(S^2 + S)\},$$

$$2D = -1.4 \text{ gauss}, \quad A = -75 \text{ gauss}.$$

The radius of Mn^{IV}, $r = 0.52$ A, is somewhat smaller than that of Ti^{IV} with 0.63 A, so either the Mn does not lie exactly in the center of the oxygen octahedron or the octahedron is distorted. A crystalline field of at least axial symmetry in

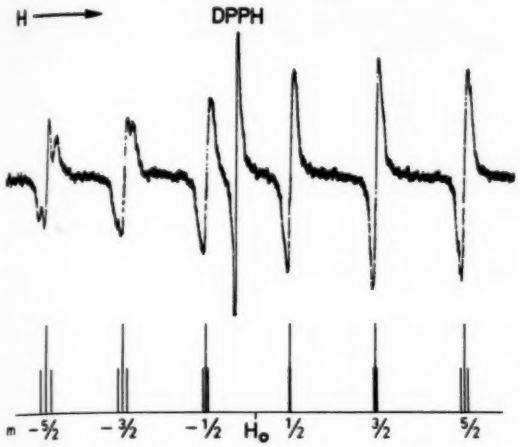

FIG. 1. Electron paramagnetic resonance spectrum of Mn^{IV} in $SrTiO_3$ at 300°K and 3.2 cm wavelength for the dc magnetic field parallel to a [100] direction. The theoretical line positions are calculated for $g = 1.994$, $A = -75$ gauss, and $2D = -1.4$ gauss.

either the [100], [010], or [001] direction results. The fine structure lines $\pm\frac{3}{2} \leftrightarrow \pm\frac{1}{2}$ arising from the [010] and [001] direction are not resolved in the spectrum and therefore also the theoretical lines are omitted.

Figure 2 shows the spectrum at liquid air temperature, again for H parallel to [100]. The lines denoted by "tetragonal part" were observed, when the $\pm\frac{1}{2} \leftrightarrow \pm\frac{3}{2}$ and $\pm\frac{3}{2} \leftrightarrow \pm\frac{5}{2}$ lines of the impurity present, Fe^{3+}, were also split[1] (in Fig. 2 only the $\pm\frac{1}{2}$ Fe^{3+} line is seen). So we conclude that this splitting is caused by that part of the crystal already in the tetragonal phase. In this phase an additional tetragonal field is superposed on the field caused by local distortion. The amplitude of the lines denoted as "cubic part," with the same position as in Fig. 1, disappear when the crystal is cooled down to 77°K. There the crystal is completely in the tetragonal phase and consists of tetragonal domains.[1]

At liquid air and liquid nitrogen temperatures we have also observed 5 lines or line groups occurring about midway between the 6 main hyperfine groups which we attribute to $|\Delta M| = 1$ and $|\Delta m| = 1$ "forbidden" transitions. These lines have been recorded for constant magnetic field directions parallel to the [100], [110], and [111] axes.

The author has profited from discussions with Professor W. Low and Dr. R. Lacroix. He thanks Dr. A. Linz for sending him the MnO_2-doped crystal and Mr. J. Sierro for orienting and cutting the samples carefully and assisting in some of the experiments.

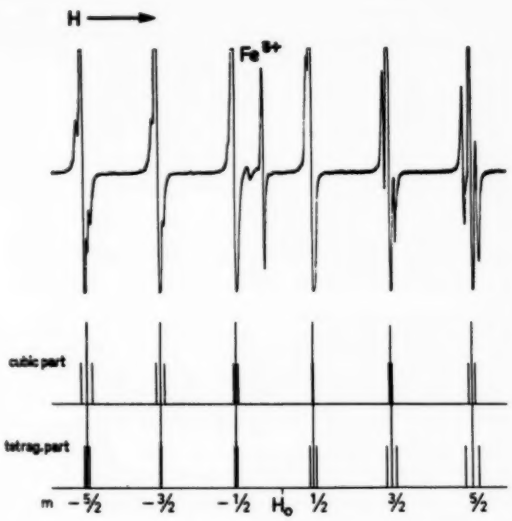

FIG. 2. Paramagnetic resonance spectrum of Mn^{IV} in $SrTiO_3$ at liquid air temperature; $H \parallel [100]$. The $\pm\frac{1}{2}$ line of the impurity present, Fe^{3+}, is also seen. The theoretical position of the lines denoted by "cubic part" are the same as in Fig. 1. The ones denoted by "tetragonal part" have been calculated with $2D = 2.1$ gauss.

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†Now with Battelle Memorial Institute, Geneva, Switzerland.

[1]K. A. Müller, Helv. Phys. Acta 31, 173 (1958).

[2]V. M. Goldschmidt, Internationale Tabellen zur Bestimmung von Kristallstrukturen (Gebrüder Borntraeger, Berlin, 1935), Vol. 2.

[3]W. Low, Phys. Rev. 105, 793 and 801 (1957).

[4]K. A. Müller, 7ème Colloque Ampère, Arch. Sci. (Genève) 11, 150 (1958).

[5]Wertz, Auzins, Griffiths, and Orton, Trans. Faraday Soc. (to be published).

[6]G. H. Jonker and J. H. van Santen, Physica 16, 337 (1950).

[7]Baker, Bleaney, and Bowers, Proc. Phys. Soc. (London) B69, 1025 (1956).

[8]K. A. Müller (to be published).

[9]J. Owen, Proc. Roy. Soc (London) A227, 183 (1955).

MEASUREMENT OF THE ATTACHMENT OF SLOW ELECTRONS IN OXYGEN*

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(Received March 20, 1959)

The study of electron attachment in oxygen has been the subject of numerous investigations,[1-6] and is currently of great interest in connection with upper atmosphere research. In general, the measurements have been interpreted in terms of two-body attachment processes for electron energies ranging from thermal (0.04 ev) to tens of electron volts. The present measurements extend drift tube measurements into the thermal energy range where they are compared with microwave afterglow results[5] and give evidence that at low electron energies (<1 ev) the attachment occurs by a three-body process.

The present method is similar in principle to that used by Doehring.[2] A pulse of electrons is liberated from a photocathode and drifts in a uniform electric field to a collector electrode. A shutter grid[6] near the collector is used to control the flow of electrons and ions to the collector. The time at which the shutter is opened can be delayed with respect to the electron pulse in order to observe the time dependence of the negative-ion arrival at the collector.

If the electron attachment probability per unit drift distance in the field is α, then the decrease in electron density, n_e, in the pulse while moving a distance, dx, in the field direction is $dn_e = -\alpha n_e dx$. From this it can be shown that the arrival of ions at the collector is described by

$$N_i = N_i(0)\exp(\alpha w_i t), \qquad (1)$$

where w_i is the ion drift velocity, determined from the time required for the ions to cross the drift space.

In the present experiment, we measure α as a function of E/p, the ratio of the drift field to gas pressure. Results obtained in pure oxygen are shown in Fig. 1. It will be seen that for $E/p>3$ the quantity α/p depends on E/p alone, while for $E/p<2$, α/p increases linearly with p at each value of E/p. Results obtained in helium-oxygen mixtures, which extend to even lower E/p values, verify this pressure dependence down to near-thermal energies.

The quantity α is related to the attachment frequency, ν_a, by $\nu_a = \alpha w_e$, where w_e is the

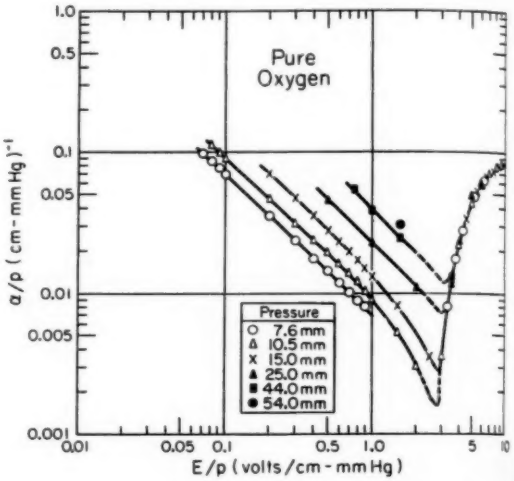

FIG. 1. Measured attachment coefficients (α/p) as a function of E/p for pure O_2 at 300°K. Drift distances of 2.5 cm and 10 cm were used.

electron drift velocity. If electrons are lost by two-body collisions involving an electron and a neutral molecule (e.g., dissociative attachment) and by three-body collisions involving the electron, the molecule, and another molecule which stabilizes the reaction,[7] we may write for the time rate of change of electron density

$$dn_e/dt = -\nu_a n_e = -(\beta n + Kn^2)n_e, \qquad (2)$$

where β is the two-body attachment coefficient, n is the gas density, and K is the three-body coefficient.

The data of Fig. 1 are converted to values of β and K as functions of the average electron energy by the use of curves of electron drift velocity[8] and average electron energy[9] versus E/p. The results are shown in Figs. 2 and 3. The two-body coefficient, β, rises rapidly with average electron energy above ~1.3 ev. As has been shown by Craggs et al.,[10] this process is the dissociative attachment reaction, $O_2 + e \rightarrow O + O^-$, observed in their beam experiments. The agreement concerning magnitude and energy depend-

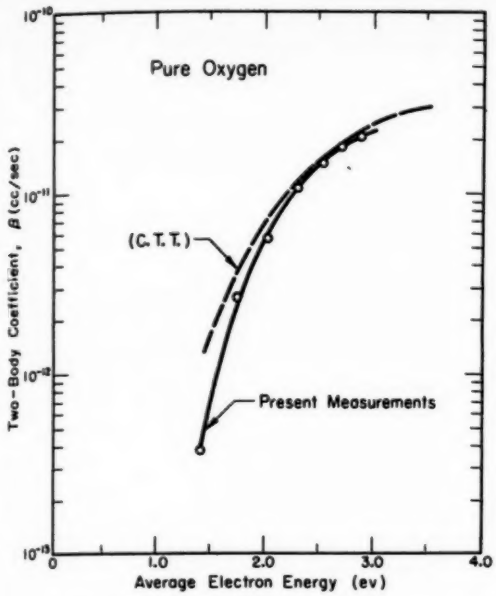

FIG. 2. Energy dependence of the two-body attachment coefficient. The dashed curve was obtained by averaging the cross sections measured by Craggs, Thorburn, and Tozer (C.T.T.) over a Druyvesteyn electron energy distribution for various average energies.

ence shown in Fig. 2 between our measurements (solid line) and an appropriate average over their data (dashed line) is within the uncertainties in the energy scale.

The three-body attachment coefficient observed at low electron energies is shown in Fig. 3. Earlier investigators appear to have missed this dependence because of large experimental errors[1] or because measurements were only made at a single pressure.[2] The data of Doehring fall within 20% of our 300°K curves from 0.7-1.0 ev. This three-body pressure dependence was recently independently identified by Hurst and Bortner,[11] who obtained data in N_2-O_2 mixtures down to 0.35 ev. Their K values are approximately 10% lower than ours over the common energy range.

The helium-oxygen data[12] indicate a maximum in the K value for oxygen at average electron energies of ~0.08 ev for T_{gas} = 300°K (0.04 ev) and at ~0.1 ev for T_{gas} = 77°K (0.01 ev). The "thermal value" ($T_{gas} = T_{electron} = 300$°K) is approximately $K = 2.8 \times 10^{-30}$ cm^6/sec. This value leads to an attachment rate ~1000 times larger than

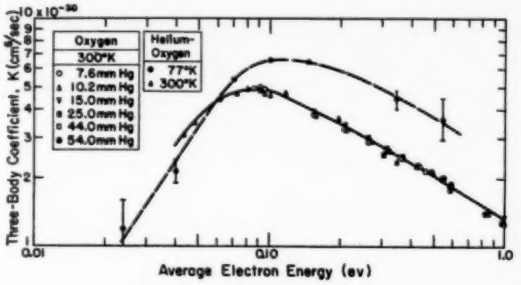

FIG. 3. Energy dependence of the three-body attachment coefficient for oxygen at 300°K and 77°K. The solid points were obtained from measurements of attachment coefficients in oxygen-helium mixtures containing 1-5% of oxygen.

that deduced from microwave studies of the oxygen afterglow[5] made at ~10 mm Hg pressure. The microwave results indicate a two-body attachment process. It has been suggested that the low microwave attachment rate is the result of substantial detachment of electrons during the afterglow occurring when vibrationally excited oxygen molecules produced during the microwave discharge strike negative ions.[13] In this case the final decay of electron density could occur at the rate of depletion of the vibrationally excited neutral molecules, leading to a low apparent attachment rate and a two-body type pressure dependence. Recent preliminary microwave afterglow studies[14] aimed at reducing this effect through the use of helium-oxygen mixtures yield attachment rates consistent with the drift tube studies.

A detailed discussion of the experimental results in terms of possible attachment processes and available theory[7] is rather lengthy and is deferred to a later article.

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*This work has been supported in part by an Air Force Special Weapons Center contract.

[1]N. E. Bradbury, Phys. Rev. 44, 883 (1933).

[2]A. Doehring, Z. Naturforsch. 79, 253 (1952).

[3]P. Herreng, Cahiers phys. 38, 7 (1952).

[4]D. S. Burch and R. Geballe, Phys. Rev. 106, 183 (1957).

[5]M. A. Biondi, Phys. Rev. 84, 1072 (A) (1951); E. Holt, Bull. Am. Phys. Soc. Ser. II, 4, 112 (1959); M. C. Sexton (private communication).

[6]For a summary of the attachment studies and of shutter designs, see L. B. Loeb, Basic Processes of Gaseous Electronics (University of California Press,

Berkeley, 1955).

[7]F. Bloch and N. E. Bradbury, Phys. Rev. 48, 689 (1935). Other types of atoms or molecules may act as the third body. Mixture experiments in gases, such as He-O_2 and N_2-O_2, give K values for He and for N_2 as third bodies which are about 250 and 50 times smaller, respectively, than those for O_2.

[8]These curves are an average of the data of H. L. Brose, Phil. Mag. 50, 536 (1925); R. A. Nielsen and N. E. Bradbury, Phys. Rev. 51, 69 (1937); R. W. Crompton (private communication); and P. Herreng, reference 3.

[9]These curves are an average of the data of R. H. Healey and J. W. Reed, The Behavior of Slow Electrons in Gases (Amalgamated Wireless, Sydney, 1951), p. 94*ff*; H. L. Brose, Phil. Mag. 50, 536 (1925); R. W. Crompton (private communication). We have taken the average energy to be equal to 1.5 times the experimentally determined values of D/μ.

[10]Craggs, Thorburn, and Tozer, Proc. Roy. Soc. (London) A240, 473 (1957).

[11]T. E. Bortner and G. S. Hurst, Health Phys. 1, 39 (1958); G. S. Hurst and T. E. Bortner, Proceedings of the International Congress of Radiation Research, August, 1958 (to be published), and Phys. Rev. (to be published).

[12]Each point shown is derived from the slope of a plot of resultant K as a function of the fractional oxygen concentration at very low oxygen concentrations, e.g., 1-5%.

[13]J. D. Craggs, Third International Conference in Ionization Phenomenon in Gases, Venice, Italy, June, 1957 (The Italian Physical Society, October, 1957), p. 207.

[14]W. A. Rogers and M. A. Biondi (private communication).

PROPOSAL FOR A METHOD OF POLARIZING 1-Mev DEUTERONS

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(Received March 5, 1959)

The purpose of this note is to point out that the elastic scattering of deuterons from He^4 can yield a rather large deuteron polarization with a high cross section. The differential cross section in d-He^4 elastic scattering has been measured to 3% or better in the region of 1-Mev incident deuteron energy.[1] The scattering in this region is characterized by a resonance at 1.070 Mev which corresponds to the formation of the first excited state of Li^5, to which is given the assignment $J=3^+$.[2] If one uses the phase-shift analysis as calculated from these data assuming a single resonance level with $J=3$, $L=2$,[2] then one can construct the tensor invariants of the polarization for the scattered deuteron beam as a function of energy.[3] This is done by writing the scattered wave for incident spin projection m_s in the form

$$\psi(m_s)=\sum_{m_s'} a_{m_s'}{}^{m_s}\chi_{m_s'}{}^{s}, \qquad (1)$$

where $\chi_{m_s'}{}^{s}$ is an eigenvector of S_z, and $a_{m_s'}{}^{m_s}$ is a coefficient involving the phase shifts and spherical harmonics. Here we choose the z axis in the incident direction, and the y axis normal to the scattering plane. Then the density matrix for the scattered beam is

$$\langle m_s'|\rho|m_s''\rangle=(2s+1)^{-1}\sum_{m_s} a_{m_s'}{}^{m_s}a_{m_s''}{}^{m_s*}, \qquad (2)$$

normalized such that $\mathrm{Tr}\{\rho\}=d\sigma/d\Omega=I_0$. Following reference 3, we express the density matrix in terms of the unit matrix plus five independent tensor invariants:

$$\rho=\sum_{JM}A_{JM}T_{JM}, \qquad (3)$$

where T_{JM} transforms under rotation of the quantization axis like the spherical harmonic $Y_M{}^J(\theta,\phi)$. The expectation values $\langle T_{JM}\rangle$ can be calculated from the general rule

$$\langle T_{JM}\rangle=I_0{}^{-1}\,\mathrm{Tr}\{\rho\,T_{JM}\}, \qquad (4)$$

once the density matrix is known in terms of the phase shifts.

Figure 1 shows the cross section, I_0, and Figs. 2 and 3 give the tensor invariants for the scattered wave, with the z axis as chosen above, as a function of the incident deuteron energy in the region of the resonance. Coulomb forces are included in the calculation. The definitions of the invariants as shown on the figures are the same as in reference 3. The curves are reliable only in the region near the resonance energy, because the approximation $\tan\beta_2{}^3=\Gamma/[2(E_0-E)]$ was used for the resonant phase shift $\beta_2{}^3$. $\langle T_{10}\rangle$ of course, vanishes because of parity conservation, and $\langle S_x\rangle$ vanishes because of the choice of the y axis. Thus $i\langle T_{11}\rangle=(\sqrt{3}/2)\langle S_y\rangle$.

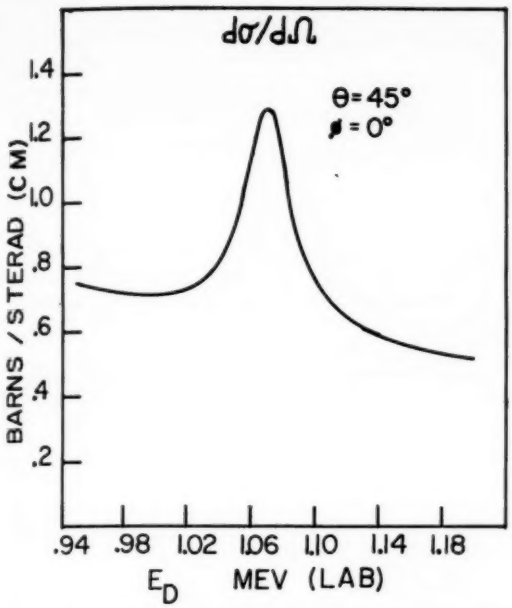

FIG. 1. Differential cross section in the center-of-mass system for d-He^4 elastic scattering as a function of incident deuteron energy in the lab system, calculated from the phase-shift analysis of reference 2.

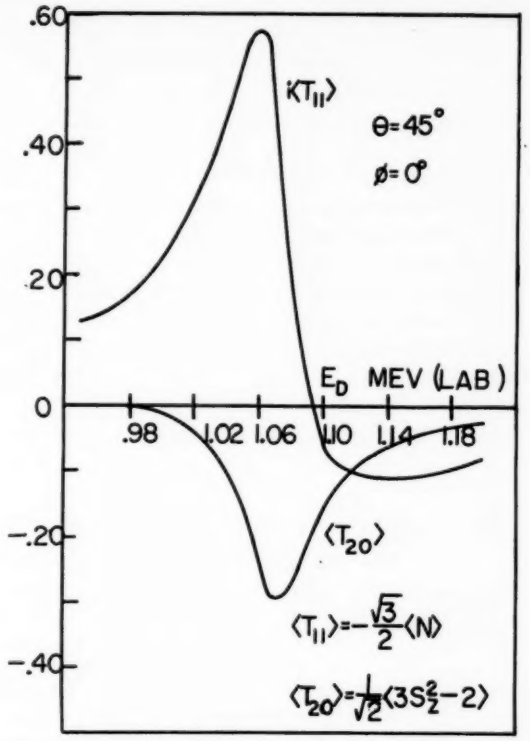

FIG. 2. Tensor invariants of the deuteron polarization in d-He^4 elastic scattering as defined in reference 3, calculated from the phase-shift analysis of reference 2. Since $N=S_x+iS_y$, in no reaction can $i\langle T_{11}\rangle$ exceed $\sqrt{3}/2$.

In the center-of-mass system this term has angular dependence proportional to $\sin\theta\cos\theta$, which is a maximum at $\theta=45°$. Due to interference effects, $\langle T_{11}\rangle$ and I_0 do not peak at the same energy. At 45°, $\langle T_{11}\rangle$ is 10 kev lower, and the difference is not eliminated by varying θ in the region of 45°. Because of their behavior near resonance and their small magnitude, both $\langle T_{22}\rangle$ and $\langle T_{21}\rangle$ may be neglected.

In order to consider the effect of this polarization on a second reaction, the axis of quantization should be rotated so that the tensor components are described in terms of the new incident direction as the z direction. Since the orientation of the deuteron spin vector in its own center-of-mass system is not affected by adding an arbitrary velocity to the deuteron, the desired angle of rotation places the quantization axis parallel to the lab momentum vector after the first scattering. For deuterons incident on He^4, $\theta_{c.m.}=45°$ corresponds to $\theta_{lab}=30°$. The transformation into the center-of-mass system of the second reaction does not change the relative orientation of the z and quantization axes. Thus Eq. (2.8) of reference 3 can be used

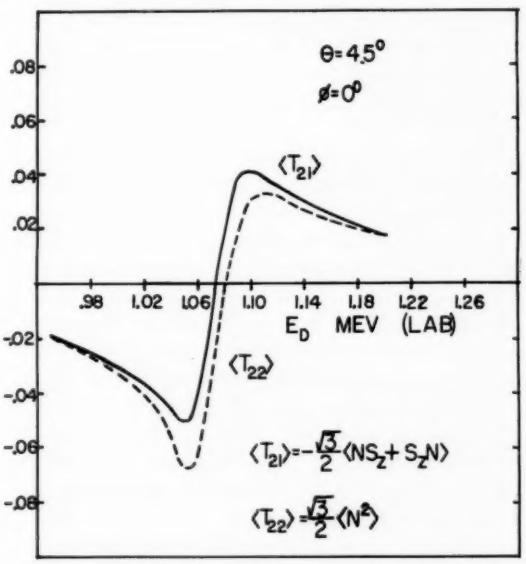

FIG. 3. Tensor invariants of the deuteron polarization in d-He^4 elastic scattering as defined in reference 3.

directly, after the rotation is performed on $i\langle T_{11}\rangle$ and $\langle T_{20}\rangle$.[4] The result is as follows:

$$I(\theta,\phi) = I_0(\theta) + \tfrac{1}{8}\langle T_{20}\rangle A(\theta) + [\tfrac{3}{8}(2)^{1/2}\langle T_{20}\rangle B(\theta) + i\langle T_{11}\rangle C(\theta)]\sin\theta\cos\phi + \tfrac{1}{16}(6)^{1/2}\langle T_{20}\rangle D(\theta)\sin^2\theta\cos 2\phi, \quad (5)$$

where I_0 is the unpolarized cross section; A, B, C, and D are polynomials in $\cos\theta$, and are dependent upon the reaction matrix elements. Because of the large magnitude of $i\langle T_{11}\rangle$ after the initial scattering and the low energy, one would expect the $\cos\phi$ term in the asymmetry to dominate; the "right-up" type of asymmetry characteristic of spin one would probably be much smaller. Professor W. M. MacDonald has suggested to the author that such deuterons may be a useful tool in investigating (d,p) and (d,n) reactions. Dr. A. S. Langsdorf has pointed out that polarized deuterons of higher energy may be obtained by accelerating He^4 nuclei on a deuterium target, rather than the inverse.

[1]Galonsky, Douglas, Haeberli, McEllistrem, and Richards, Phys. Rev. 98, 586 (1955).
[2]A. Galonsky and M. T. McEllistrem, Phys. Rev. 98, 590 (1955).
[3]W. Lakin, Phys. Rev. 98, 139 (1955).
[4]For the rotation matrices see, for example, M. E. Rose, Elementary Theory of Angular Momentum (J. Wiley and Sons, New York, 1957), Chap. IV.

MAGNETIC PERTURBATION ON CIRCULAR POLARIZATION OF EXTERNAL BREMSSTRAHLUNG

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(Received February 17, 1959)

It is well known that the external bremsstrahlung (EB) produced by β-rays is circularly polarized. The degree of circular polarization, according to theoretical predictions,[1-3] is a function of the quantum energy and attains its maximum value when the quantum has the same energy as the radiating electron. The EB polarization has been measured by some investigators[4-7] whose results appear to agree reasonably as far as concerns the energy dependence. A lack of agreement seems to exist regarding the dependence of the polarization on the atomic number of the absorber. A strong Z dependence has been reported by Cohen et al.[6] but it was not confirmed by Galster and Schopper.[5] Recently we[8] have found that the atomic number of the target affects the EB polarization to an extent which seems to substantiate the calculations of Neamtan[9] on electron depolarization in matter.

With a view to obtaining further information on the influence of matter on EB polarization, we have investigated the circular polarization-energy relation for EB quanta, by using a magnetized iron target as β absorber. The circular polarization of γ-rays was analyzed through Compton scattering with polarized electrons available in magnetized iron. The experimental apparatus and procedure were the same as previously described.[7-8] The source of β-rays (Y^{90}) was placed just behind the target (about 8 mm thick) which was magnetized perpendicularly to the polarimeter axis ($B = 16\,000$ gauss).

The measurements consisted of several 6-minute counting runs, one for each opposite polarimeter field direction, made alternately with magnetized and unmagnetized targets. Every run showed that the asymmetry in the counting rate for opposite polarimeter field direction was lower when the target was magnetized. Taking into account the correction factor of any instrumental effect, and the efficiency of the polarimeter, we obtain the polarization-energy relation for EB quanta produced in the unmagnetized target. The relation for EB quanta produced in the magnetized target is obtained from the preceding relation by multiplying by the ratio of the measured asymmetries. This is right, in view of the fact that the interesting spectral distributions for the two magnetization states of the target have identical shapes. The two relations are shown in Fig. 1. For each curve a total of 6×10^7 pulses were counted. The quoted errors are only statistical.

It is noteworthy that at low energies ($E<1.4$

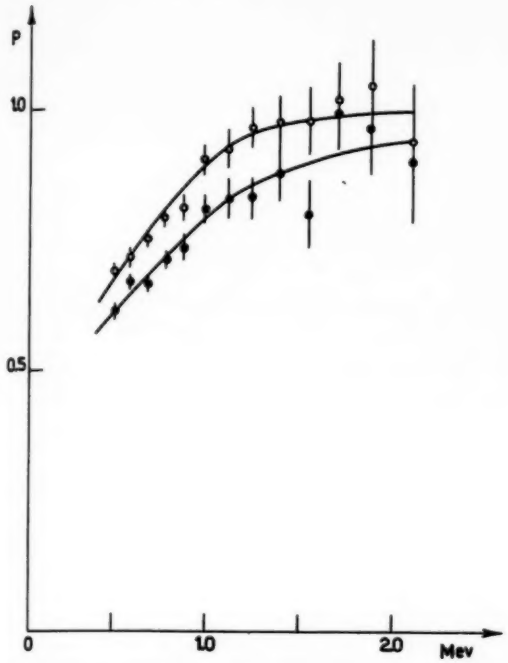

FIG. 1. Circular polarization of external bremsstrahlung quanta from Y^{90} β-rays, produced in unmagnetized (upper curve) and magnetized iron (lower curve).

Mev) the percentage decrease of polarization appears to be nearly constant against E and equal to about 10%. At higher energies the statistical errors prevent a significant conclusion, but the results are not in conflict with the reasonable assumption that the magnetic perturbation vanishes at the high-energy end of the spectrum.

Although a quantitative account of the effect cannot be given at present, the existence of a magnetic perturbation on circular polarization of EB is not surprising. It must not be forgotten that in magnetized iron the β particles are subject to strong inhomogeneous electric and magnetic fields varying on a microscopic scale, which are able to produce electron depolarization. If this hypothesis is correct, the superficial magnetic structure (thickness less than 0.2 mm) of the iron target should play an important role.

[1]K. W. McVoy, Phys. Rev. 106, 828 (1957); 110, 1484 (1958).
[2]C. Fronsdal and H. Überall, Phys. Rev. 111, 580 (1958).
[3]H. Banerjee, Phys. Rev. 111, 532 (1958).
[4]Goldhaber, Grodzins, and Sunyar, Phys. Rev. 106, 828 (1957).
[5]S. Galster and H. Schopper, Phys. Rev. Lett. 1, 330 (1958).
[6]Cohen, Wiener, Wald, and Schmorak, in the Proceedings of the Rehovoth Conference on Nuclear Structure (North Holland Publishing Company, Amsterdam, 1958), p. 404.
[7]A. Bisi and L. Zappa, Phys. Rev. Lett. 1, 332 (1958).
[8]A. Bisi and L. Zappa, Nuclear Phys. (to be published).
[9]We thank Dr. S. M. Neamtan for making his calculations available to us before publication.

S-WAVE DETECTOR OF DEUTERON POLARIZATION AND 14-Mev POLARIZED-NEUTRON SOURCE

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(Received March 9, 1959)

Recently a great deal of interest[1] has been expressed in the development of polarizing ion sources for use in accelerators. In this connection a convenient analyzer for the resulting polarized beams would be most useful. It is well known that in both scattering and reactions induced by polarized s-wave protons the angular distribution of the scattered protons, or of any reaction product, is isotropic[2] just as they would be for unpolarized s-wave protons.[3] The purpose of this Letter is to point out that the fact that isotropy does not necessarily follow for a reaction induced by polarized s-wave deuterons[4] may enable one to use, for example, the $T(d,n)He^4$ reaction as a sensitive detector of the polarization of a deuteron beam. This reaction will then also serve as a source of polarized, 14-Mev neutrons.

The different results obtained for proton and deuteron beams may be ascribed to the fact that

the spin projection has only two possible values for the proton, but three for the deuteron. We have for the d-T resonance ($E_{max}=107$ kev, $l_d=0$, $J=3/2^+$, $l_n=2$)[5] the following partial cross sections[6]:

$$\left(\frac{d\sigma}{d\Omega}\right)^{+1}_{3/2,\,1/2}=\left(\frac{d\sigma}{d\Omega}\right)^{+1}_{1/2,\,-1/2}=\left(\frac{d\sigma}{d\Omega}\right)^{-1}_{-3/2,\,-1/2}=\left(\frac{d\sigma}{d\Omega}\right)^{-1}_{-1/2,\,1/2}$$
$$=\tfrac{1}{2}\left(\frac{d\sigma}{d\Omega}\right)^{0}_{1/2,\,-1/2}=\tfrac{1}{2}\left(\frac{d\sigma}{d\Omega}\right)^{0}_{-1/2,\,1/2}=3K\sin^2\theta\cos^2\theta,$$

$$\left(\frac{d\sigma}{d\Omega}\right)^{+1}_{3/2,\,-1/2}=\left(\frac{d\sigma}{d\Omega}\right)^{-1}_{-3/2,\,1/2}=3K\sin^4\theta,$$

$$\left(\frac{d\sigma}{d\Omega}\right)^{+1}_{1/2,\,1/2}=\left(\frac{d\sigma}{d\Omega}\right)^{-1}_{-1/2,\,-1/2}=\tfrac{1}{2}\left(\frac{d\sigma}{d\Omega}\right)^{0}_{1/2,\,1/2}=\tfrac{1}{2}\left(\frac{d\sigma}{d\Omega}\right)^{0}_{-1/2,\,-1/2}$$
$$=\tfrac{1}{3}K(9\cos^4\theta-6\cos^2\theta+1),$$

where $K=|U|^2/16k^2$ is independent of angle, U is the relevant element of the collision matrix, and k is the deuteron wave number. The superscripts on the cross sections are the magnetic quantum numbers of the deuterons; the subscripts are those of the channel spins referred to the beam axis,[7] the first subscript for the d-T channel (channel spin 1/2 does not contribute to this resonance reaction since $J=3/2$ and $l_d=0$) and the other for the n-He^4 channel. The cross section for a deuteron beam polarized with $m_d=+1$ is

$$\sum_{a,b}\left(\frac{d\sigma}{d\Omega}\right)^{+1}_{a,\,b}=\tfrac{2}{3}K(3\sin^2\theta+2);$$

if $m_d=-1$ we get the same result, and if $m_d=0$ we get $\frac{4}{3}K(3\cos^2\theta+1)$. The differential cross section for an unpolarized beam, which is the average of the above three cross sections, is $8K/3$; isotropic, as it must be.[3] The equality of the $m_d=\pm1$ cross sections is a consequence of reflection invariance of the nuclear interaction. Similarly, for a proton beam $m_p=+\frac{1}{2}$ and $-\frac{1}{2}$ give equal cross sections. These are isotropic for s-waves, since an unpolarized beam, which can be represented as an equal mixture of $m_p=\pm\frac{1}{2}$, gives isotropy.[3]

Thus, we see that the angular distribution of the neutrons or alpha particles in the d-T reaction is determined by the alignment (mean value of m_d^2) rather than the polarization (mean value of m_d). If one were attempting experimentally to produce a pure $m_d=+1$ beam, for example, the major impurity would most likely be $m_d=0$. Then the alignment and polarization would be equal. Figure 1 is a set of angular distributions resulting from various mixtures of $m_d=+1$ and 0 in the deuteron beam.

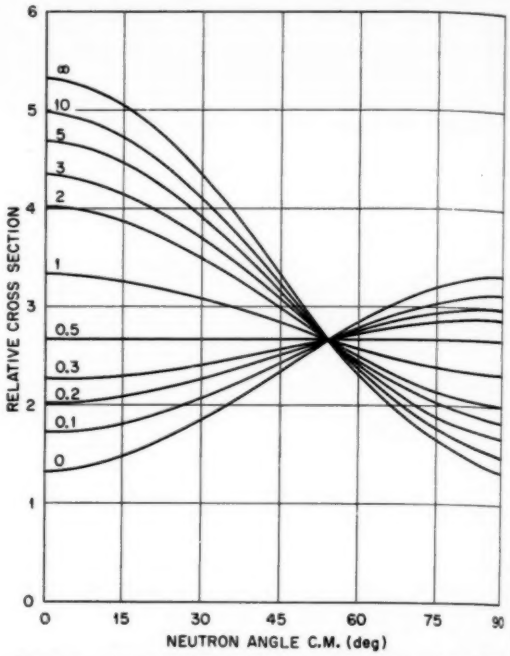

FIG. 1. Angular distributions of neutrons from the $T(d,n)He^4$ resonance reaction for polarized beams of deuterons containing a mixture of $m_d=+1$ and 0. The label on each curve is the ratio of the $m_d=0$ component to the $m_d=+1$ component.

The main advantage of attempting to produce beams of polarized deuterons rather than polarized protons is that s-waves and, therefore, low voltages are sufficient to produce anisotropy. In our $T(d,n)He^4$ example the cross section has its maximum at only 107 kev. In addition, this cross section at the maximum is very large (5 barns for an unpolarized beam) and the width is 140 kev. This scheme obviates the need for a pressurized electrostatic generator. With a low-voltage, open-air accelerator there need be no limitation on the size of the polarizing apparatus and the easy accessibility of such apparatus has many obvious advantages. If convenient, the polarity of the accelerator may even be reversed so that the polarizer and ion source are at ground potential. As an example of the rapidity of determination of the polarization, consider the following conditions for a pure $m_d=+1$ beam, where the neutrons from a thick Zr-T target are detected at 90°: deuteron energy = 200 kev, deuteron current = 1 millimicroampere, detector solid angle = 1% of sphere (half-angle = 12°), and

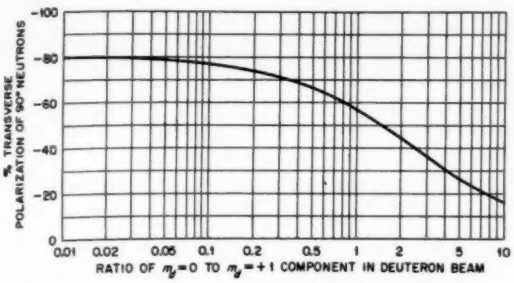

FIG. 2. Polarization of neutrons produced at 90° in the $T(d,n)He^4$ resonance reaction by a polarized deuteron beam. The direction of polarization is opposite to the beam direction in the scattering plane, hence transverse to the neutron direction.

detector efficiency = 20%. The resulting count rate is ~300/sec.

Reference to the partial cross sections listed above shows that the 14-Mev neutrons produced in the d-T reaction are, in general, polarized. The $m_d = \pm 1$ deuterons produce equal and opposite neutron polarizations and the $m_d = 0$ deuterons no polarization. Figure 2 shows the polarization of the neutrons at 90° as a function of the contamination of an $m_d = +1$ beam by $m_d = 0$. The direction of polarization is opposite to the beam direction in the scattering plane, hence transverse to the neutron direction. It is, of course, essential to the use of the neutrons in a subsequent nuclear process that the neutron polarization be transverse. By examination of the neutron angular distribution and polarization, it therefore appears that the complete polarization state of the deuteron beam can be determined.

It should be pointed out that the analogous $He^3(d,p)$ reaction may be an even more convenient one for determining the polarization state of the deuteron. This reaction would, in addition, produce 14-Mev polarized protons.

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[1]Clausnitzer, Fleischmann, and Schopper, Z. Physik 144, 336 (1956); E. K. Zavoiskii, J. Exptl. Theoret. Phys. U.S.S.R. 32, 408, 731 (1957) [translation: Soviet Phys. JETP 5, 338, 603 (1957)]; R. L. Garwin, Rev. Sci. Instr. 27, 374 (1958); and L. Madansky and G. E. Owen, Phys. Rev. Lett. 2, 209 (1959).

[2]L. Wolfenstein, Phys. Rev. 75, 1664 (1949).

[3]Eisner, Sachs, and Wolfenstein, Phys. Rev. 72, 680 (1947); 73, 528 (1947). C. N. Yang, Phys. Rev. 74, 764 (1948).

[4]It has already been shown independently that polarized S-wave deuterons may lead to anisotropy; L. J. B. Goldfarb, Nuclear Phys. 7, 622 (1958).

[5]F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. 27, 77 (1955), pp. 78-79.

[6]R. G. Sachs, Nuclear Theory (Addison-Wesley Press, Cambridge, 1953), Chap. 10, especially Eqs. (10-12) and (10-54).

[7]Because this reaction picks out of the deuteron beam only the S-wave part, which has no preferred direction, the angular dependences of the partial cross sections are really referred to the alignment axis, chosen here to coincide with the beam axis. For a different alignment axis the results may be obtained by rotating the angular distributions presented above through the angle between the two axes. In this case the differential cross sections will depend on both the polar and azimuthal angles. If the polarization state of the deuteron beam is too complex to be described solely in terms of one alignment axis, then the calculation must be altered.

RESONANCES IN THE π-HYPERON SYSTEM

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(Received March 30, 1959)

The scattering matrix for the charge symmetric π-Σ-Λ interaction[1] has been investigated in the Chew-Low approximation. The Σ-Λ relative parity was assumed to be even. In the case of global symmetry ($g_{\Sigma\Sigma\pi} = \pm g_{\Sigma\Lambda\pi}$) we obtain the familiar Chew-Low equations for the π-nucleon system; the π-nucleon states of isotopic spin $T = \frac{3}{2}$ and $T = \frac{1}{2}$ corresponding now to the π-hyperon states $T = 2$ and $T = 0$, respectively. The scattering amplitude for the two $T = 1$ states ($\Sigma\pi$ and $\Lambda\pi$) can be written as a linear combination of the amplitudes in the $T = 2$ and $T = 0$ states. The effective range in the $T = 2$, $J = \frac{3}{2}$ state is positive, and if the coupling constant and cutoff energy are comparable to the π-nucleon values we get the analog of the famous π-nucleon $T = \frac{3}{2}$, $J = \frac{3}{2}$ resonance. In the limits $g_{\Sigma\Lambda\pi} \ll g_{\Sigma\Sigma\pi}$ and $g_{\Sigma\Lambda\pi} \gg g_{\Sigma\Sigma\pi}$, the effective range also turns out to be positive in the $T = 2$, $J = \frac{3}{2}$ state. In addition the effective range is positive in the $T = 0$, $J = \frac{1}{2}$ state for the case $g_{\Sigma\Lambda\pi} \ll g_{\Sigma\Sigma\pi}$ and in the $T = 1$, $J = \frac{1}{2}$ ($\Sigma\pi$) and $T = 1$, $J = \frac{3}{2}$ ($\Lambda\pi$) states in the case $g_{\Sigma\Lambda\pi} \gg g_{\Sigma\Sigma\pi}$.

This note was prompted by the recent analysis of the K^--p scattering experiments by Matthews and Salam.[2] They suggest that near $T_K \approx 25$ Mev the interaction takes place through a $J=\frac{1}{2}$ resonance. The experiments do not suffice to determine the isotopic spin at resonance. We wish to point out that, while a $J=\frac{1}{2}$ resonance at these energies is not consistent with global symmetry, it can be explained by assuming either $g_{\Sigma\Lambda\pi} \ll g_{\Sigma\Sigma\pi}$ if $T=0$ or $g_{\Sigma\Lambda\pi} \gg g_{\Sigma\Sigma\pi}$ if $T=1$. The limit $g_{\Sigma\Lambda\pi} \ll g_{\Sigma\Sigma\pi}$ is very hard to reconcile with the near equality of the Λ-nucleon and nucleon-nucleon forces, and with the K^- capture experiments in deuterium which indicate that there exists a large Σ-Λ exchange. The possibility $g_{\Sigma\Lambda\pi} \gg g_{\Sigma\Sigma\pi}$ is compatible with these results. These remarks are strictly valid only if the K-nucleon-hyperon interaction is small compared to the π-hyperon interaction. A more detailed investigation including the K interaction is in progress.

I wish to thank Dr. R. Capps and Professor T. Kinoshita for several enlightening discussions and Professor H. Bethe for his constant encouragement and advice.

[1]M. Gell-Mann, Phys. Rev. 106, 1296 (1957).
[2]P. T. Matthews and A. Salam, Phys. Rev. Lett. 2, 226 (1959).

EVIDENCE FROM PHOTOPRODUCTION FOR A PSEUDOSCALAR K^+ MESON*

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It has been suggested recently by Taylor[1] that the extrapolation procedure used[2] to determine the pion-nucleon coupling constant from pion photoproduction angular distributions could be applied to K-meson photoproduction data to obtain the parity of the K^+ meson. This suggestion is based on the fact that the sign of the residue of the meson current pole is positive for pseudoscalar K mesons and negative for scalar K mesons.

It is easy to see[3,4] that even if the mass difference between the nucleon and the hyperon is taken into account, the above qualitative statement continues to hold. Furthermore, one can add the quantitative statement that for the same value of the coupling constant the absolute value of the residue for the scalar case is larger than for the pseudoscalar case by a huge factor which, for the associated production of K^+ and Λ^0, turns out to be about 18.5.

The above statements are partially equivalent to the well-known fact that under usual circumstances scalar photoproduction is dominated by the direct-interaction term, while in pseudoscalar photoproduction at low energies the direct-interaction term plays a relatively small role. It has been shown,[3] however, that if one takes into account the anomalous magnetic moments of the fermions involved in the reaction, the distinction between the general features of the angular distribution of scalar and pseudoscalar photoproduction becomes much less pronounced. In fact, it seems impossible to decide from the qualitative features of the angular distribution whether the particle produced is scalar or pseudoscalar. The extrapolation procedure used in this note quantitatively separates out the direct-interaction (meson current) term, and hence the ambiguity due to the anomalous magnetic moments and other uncertainties, which are contained only in the nucleon current terms, disappear.

Experimental data on K-meson photoproduction angular distributions are still scarce. In the energy interval of 1000-1010 Mev, however, there are now about ten measurements of differential cross section for the associated production of K^+ and Λ^0. The data are shown in Table I. Since these data were taken at such low energies, the distance of extrapolation[2] is quite large, the pole being at $\cos\theta = 2.70$. At the same time, however, on account of the low energies, one can attempt to use low-order polynomials for the extrapolation, since it might be assumed that only S (and perhaps P) states play a significant role beside the meson current term. This assumption, which is made in the analysis to follow, is perhaps on less secure grounds here than it would be in the case of pion photoproduction,

Table I. Experimental data of $K^+ - \Lambda^0$ photoproduction at 1000-1010 Mev as used in the present analysis, in units of 10^{-31}cm^2/sterad.

$\theta_{c.m.}$ (deg)	$d\sigma/d\Omega$	Reference
26	1.32±0.14	a
28.5	1.55±0.23	b
42	1.61±0.20	c
44	1.32±0.07	a
68	1.40±0.10	a
72	1.53±0.09	b, c
85	1.23±0.20	a
89	1.26±0.19	a
90	1.46±0.23	c
116	1.46±0.29	a

[a]McDaniel, Silverman, Wilson, and Cortellessa, Phys. Rev. Lett. 1, 108, 263 (E) (1958).
[b]Brody, Wetherell, and Walker (to be published).
[c]P. L. Donoho and R. L. Walker, Phys. Rev. 112, 981 (1958).

since in the process involving K mesons pions may also play an important role.

If only S waves are significant, the extrapolating polynomial is of second order. If one also takes into account S-P interference, a third-order polynomial is required.

Figure 1 shows the data of Table I, multiplied by the pole denominator. Also shown are the first-, second-, and third-order polynomials obtained from these data. The mathematical details of the fitting are given in Table II. One can see that the statistical improvement of the third-order polynomial over the second-order polynomial is slight. One would tend to disregard the fourth-, fifth-, and higher-order polynomials, since the statistical significance of fits with less than five degrees of freedom is very questionable. It is somewhat peculiar that the χ^2 values are considerably smaller than the numbers of degrees of freedom. This might indicate that the experimental errors have been slightly overestimated.

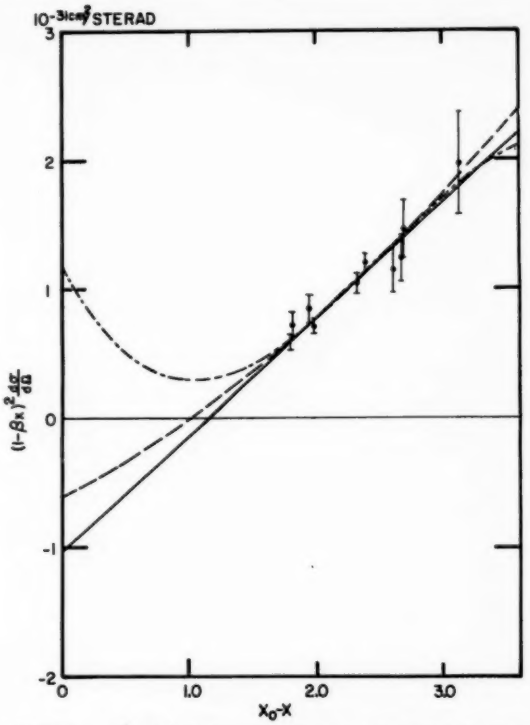

FIG. 1. $K^+ - \Lambda^0$ photoproduction angular distribution data at 1000-1010 Mev, multiplied by $(1-\beta x)^2$, vs $(x_0 - x)$, where $x = \cos\theta$ (θ being the production angle in the center-of-mass system) and $x_0 = 2.70$ is the position of the pole. The extrapolating polynomials are also shown, the solid line being the linear curve, the dashed line the second-order curve, and the dash-dot line the third-order curve.

Figure 2 shows the residue as a function of the order of the polynomial, together with the residues for scalar and pseudoscalar mesons with

Table II. Values of the statistical parameters for the extrapolation of K^+-meson photoproduction angular distribution. Detailed discussion of these parameters is given by Taylor et al.[a]

Order of polynomial	1	2	3	4	5
χ^2	6.291	6.217	6.173	4.936	3.940
Number of degrees of freedom	8	7	6	5	4
Residue, 10^{-31}cm^2/sterad	-1.03	-0.61	1.19	65.5	-1488
Error on the residue	0.19	1.48	8.84	58.0	1548
F	0.083	0.043	1.255	1.010	
F probability	22%	15%	70%	60%	

[a]See reference 2.

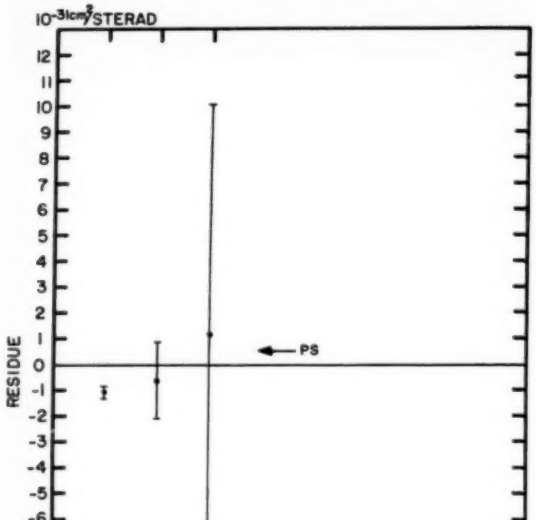

FIG. 2. The residue obtained from extrapolating $K^+ - \Lambda^0$ photoproduction angular distribution at 1000-1010 Mev, vs the order of the extrapolating polynomial. The values of the residue for scalar (S) and pseudoscalar (PS) mesons with $g^2 = 1$ are also given.

$g^2 = 1$. The residues are given by the expression[3]

$$\tfrac{1}{4} C g^2 \alpha \lambda\!\!\!^{-2} q k^{-3} W^{-2} (1 - \beta^2), \qquad (1)$$

where α is the fine structure constant, $\lambda\!\!\!^{-}$ the reduced Compton wavelength of the K meson, q its momentum, k the photon momentum, β the K-meson velocity, and W the total energy of the system, all in the center-of-mass system. The constant C is

$$4M'M - [\mu^2 - (M' - M)^2] \text{ for scalar mesons,}$$
$$\mu^2 - (M' - M)^2 \text{ for pseudoscalar mesons,} \qquad (2)$$

where μ is the K-meson mass, M' the mass of the Λ^0, and M the mass of the proton.

One can see from Fig. 2 that if the K meson is pseudoscalar it would be predicted to have a coupling constant of the order of unity, while if it is scalar its coupling constant would be more of the order of 0.1. Since from other data the evidence tends to be that the coupling constant in either case should be of the order of unity,[5,6] the present analysis can be taken as evidence for negative parity for the K^+ meson.

The great virtue of the present analysis is that it is completely independent of the validity of perturbation calculations, and of the value of the anomalous magnetic moments that have plagued previous attempts[3,4,7] to extract information from K-meson photoproduction experiments. At the same time, the present conclusions are indecisive for two reasons. Firstly, the errors on the residues, even for these lowest-order polynomials, are too large. Secondly, the number of pieces of data is too small to test the somewhat ad hoc assumption that only the very lowest angular momentum state of the nucleon current term contributes significantly. The analysis shows, however, that relatively little improvement in the experimental data could give a quite decisive answer. For instance, if the errors on the individual points were reduced by a factor of 2 and if about twice as many pieces of data were available at this energy, a similar analysis could very well be unambiguous. Since such a slight improvement in the experiments is quite possible in the near future, one might hope that, former predictions[3] notwithstanding, photoproduction might be one of the first reactions to decide the parity of the K meson.

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*Work done under the auspices of the U. S. Atomic Energy Commission.

[1]John G. Taylor (unpublished).

[2]Taylor, Moravcsik, and Uretsky, Phys. Rev. 113, 689 (1959).

[3]M. Kawaguchi and M. J. Moravcsik, Phys. Rev. 107, 563 (1957).

[4]A. Fujii and R. E. Marshak, Phys. Rev. 107, 570 (1957).

[5]R. H. Dalitz, Reports of Progress in Physics (The Physical Society, London, 1957), Vol. 20, p. 163.

[6]M. Gell-Mann, 1958 Annual International Conference on High-Energy Physics at CERN, edited by B. Ferretti (CERN, Geneva, 1958), p. 162.

[7]R. H. Capps, Phys. Rev. (to be published).

NEGATIVE K MESON–NUCLEON INTERACTION AT LOW ENERGIES

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(Received March 25, 1959)

In a recent letter Matthews and Salam[1] observed that the emulsion data[2] for the total elastic scattering of K^- on protons may indicate a broad peaking in the cross section at ~25 Mev, and that this peak could be interpreted in terms of a $J=1/2$ resonance of the K^-p system. Such a resonance necessitates an attractive effective interaction between K^- and p, consistent with the rather indirect evidence of Alles et al.[3]

We wish to point out another, at present equally acceptable, explanation of the broad peaking at low energies, within the framework of the s-wave zero-range analysis.[4,5] If the K^--p interaction is assumed "repulsive,"[6] the destructive interference with the attractive Coulomb interaction will cause the integrated cross section to fall below the value in the absence of the Coulomb field. At energies above ~50 Mev this effect is negligible, but at low energies it is sufficient to cause the integral of the elastic differential cross section (suitably cut off at small angles) to behave as implied by experiment. For an "attractive" K^--p interaction the integrated cross section falls off monotonically with increasing energy even more steeply than the zero-range cross sections in the absence of the Coulomb field.

Recently we employed the R matrix formalism of Wigner and Eisenbud to include the effects of the $\bar{K}^0$-K^- mass difference in the zero-range analysis of the K^--nucleon interactions at low energies.[7] The formalism readily allows the inclusion of the long-range Coulomb interaction in the entrance channel (K^-p).

For K^- incident on protons, the elastic scattering, charge exchange, and reaction cross sections for $T=0$ and $T=1$ can be written in the following form:

$$\frac{d\sigma_{el}}{d\Omega}=\left|\frac{\eta}{2k}\csc^2(\theta/2)\exp[+2i\eta\ln\sin(\theta/2)]+\frac{C^2(A_0+A_1-2ik'A_0A_1)}{2\Delta}\right|^2, \quad (1)$$

$$\frac{d\sigma_{ex}}{d\Omega}=\frac{C^2k'}{k}\left|\frac{A_0-A_1}{2\Delta}\right|^2, \quad (2)$$

$$\tfrac{1}{2}\frac{d\sigma_{abs}^{(0)}}{d\Omega}=\frac{C^2b_0}{2k}\left|\frac{1-ik'A_1}{\Delta}\right|^2, \quad (3)$$

$$\tfrac{1}{2}\frac{d\sigma_{abs}^{(1)}}{d\Omega}=\frac{C^2b_1}{2k}\left|\frac{1-ik'A_0}{\Delta}\right|^2, \quad (4)$$

where $A_0=a_0+ib_0$, $A_1=a_1+ib_1$ are the complex scattering lengths in the absence of the Coulomb interaction,[4,7] k and k' are the wave numbers in the (K^-p) and $(\bar{K}^0n)$ channels, $\eta=e^2/\hbar v$, $C^2=2\pi\eta\times(1-e^{-2\pi\eta})^{-1}$ is the s-wave Coulomb penetration factor, and Δ is defined by

$$\Delta=1-\tfrac{1}{2}i(A_0+A_1)[k'+C^2k(1-i\tan\alpha)]-A_0A_1k'C^2k(1-i\tan\alpha). \quad (5)$$

In Eqs. (1)-(5) it has been assumed that the range of the interaction (channel radius R) is small compared to $(\hbar^2/2me^2)=41.7\times10^{-13}$ cm and that $kR\ll1$. In that case

$$\tan\alpha=(G'/F')|_R\simeq-(2\eta/C^2)[\ln(2kR)+2\gamma+\mathrm{Re}\psi(i\eta)], \quad (6)$$

where $\gamma=0.5772...$, and ψ is the logarithmic derivative of the gamma function. In the calculations presented below, R was taken as the K meson Compton wavelength ($R=0.4\times10^{-13}$ cm), but the results are insensitive to this choice.

In Eqs. (1)-(5) the assumption of zero range (A_0 and A_1 independent of energy) implies the neglect of energy variation of the elements of the R matrix and of the momenta in the reaction channels. The earlier results[4,7] can be recovered by putting $\eta=0$ and $k'=k$.

As an illustration of the effects of the Coulomb field we have calculated the elastic scattering cross section (1) with the scattering lengths determined by Dalitz[5] (his solutions A_+). A typical center-of-mass angular distribution at a laboratory momentum of 140 Mev/c is shown in Fig. 1, where attractive (repulsive) corresponds to a_0 and a_1 both positive (negative).[8]

The experimental data for total scattering cross section represent integrals over angles, excluding the immediate forward angles. For comparison with these data we integrated our angular

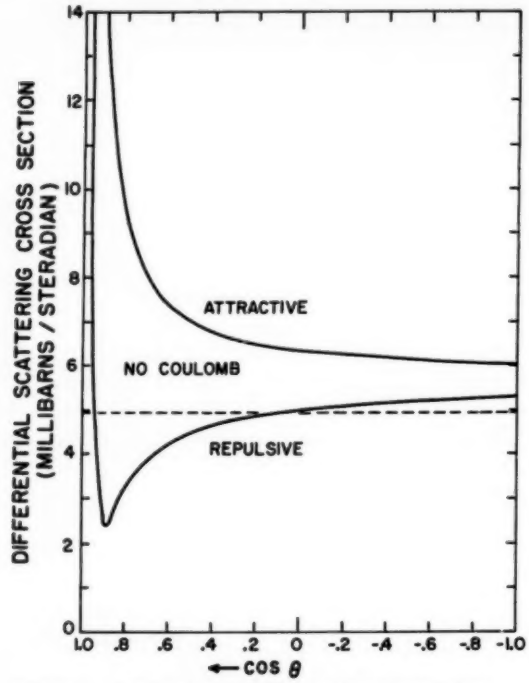

FIG. 1. Center-of-mass angular distribution for K^--p elastic scattering at 140 Mev/c laboratory momentum.

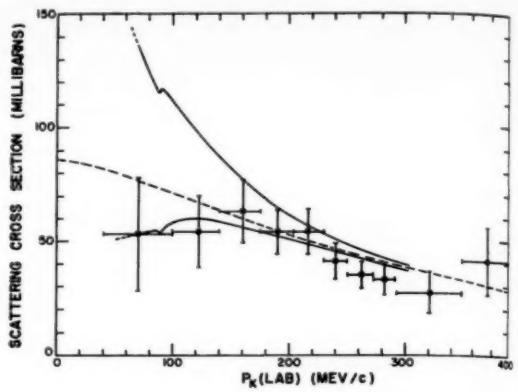

FIG. 2. Total scattering cross section as a function of laboratory momentum. The dotted curve is the zero-range result with no mass difference or Coulomb effects. The scattering lengths are $A_0 = \pm 0.28 + 0.54i$, $A_1 = \pm 1.19 + 0.22i$. The experimental points are emulsion data from reference 2.

distributions down to a forward angle determined at each energy by the criterion that the recoil proton have a laboratory momentum of at least 30 Mev/c (corresponding to a range of 5 or 6 microns in emulsions). The results for the total scattering cross section are compared with experiment[2] in Fig. 2. The effect of the destructive or constructive interference with the Coulomb amplitude is marked at low energies. At the threshold for the charge exchange reaction, small cusps appear in the theoretical curves for the elastic (and other reaction) cross sections.[7] The experimental data indicate that within the framework of the zero-range analysis the K^--p interaction is "repulsive."[9]

The comparison of theory and experiment shown in Fig. 2 can be criticized on several counts, none of which change the conclusion reached. First of all, different scattering lengths (apart from the change in sign of the real part) should be chosen to give the best over-all fit to the data for the two signs of the K^--p interaction. This would lower the "attractive" curve to give a better fit at high momenta, but its steep monotonic fall seems inconsistent with the data, independent of detailed fitting. Secondly, the variation of scattering lengths with energy due to final state interactions[10] can cause the cross sections in the absence of the Coulomb interaction to fall off less rapidly than in the zero-range approximation. While this will alter the detailed fitting, we argue that at the lowest energies such effects are not important, and the qualitative conclusion is unchanged.

Having concluded tentatively that the K^--p interaction is "repulsive" on the basis of the variation of the total elastic cross section with energy, we hasten to make the obvious remark that the differential cross section (see Fig. 1) provides a much cleaner and more direct means of determing the sign of the real part of the scattering amplitude, independent of the absolute magnitude of the cross section.

Finally we note that the sign of the K^--p amplitude at low energies has a bearing on the parity of the K meson.[11] Although there are questions regarding the handling of the unphysical regions in the dispersion relations,[12] it seems that a "repulsive" K^--p interaction leads to a rather definite conclusion. If one (a) accepts the zero-range analysis and the inference from Fig. 2 that the K^--p interaction is "repulsive," (b) assumes that the Λ and Σ have the same parity, and (c) believes in the validity of the heavy meson-nucleon dispersion relations, then the K meson is scalar, i.e., has the same parity as the Λ and Σ. If the Λ and Σ have opposite parities, no con-

clusion can be drawn about the parity of the K.

We wish to thank G. Ascoli and R. D. Hill for useful discussions about the experimental data, and R. H. Dalitz, M. Ross, and G. Shaw for conversations about the theory.

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[1]P. T. Matthews and A. Salam, Phys. Rev. Lett. 2, 226 (1959).

[2]Summarized by M. F. Kaplon in the 1958 Annual International Conference on High-Energy Physics at CERN, edited by B. Ferretti (CERN, Geneva, 1958), p. 171.

[3]Alles, Biswas, Ceccarelli, and Crussard, Nuovo cimento 6, 571 (1957).

[4]Jackson, Ravenhall, and Wyld, Nuovo cimento 9, 834 (1958).

[5]R. H. Dalitz, 1958 Annual International Conference on High-Energy Physics at CERN, edited by B. Ferretti (CERN, Geneva, 1958), p. 187.

[6]We use the terms "attractive" ("repulsive") in the usual field theoretical sense of the sign of the real part of the tangent of the phase shift at low energies being positive (negative). For a "repulsive" interaction, the question of whether the phase shift is dropping down from π (corresponding to a strong attraction) or going negative from zero (corresponding to a repulsion) can only be decided by the higher energy behavior (e.g., the sign of the effective range).

[7]J. D. Jackson and H. W. Wyld, Nuovo cimento (to be published).

[8]Dalitz showed that a_0 and a_1 must have the same sign.

[9]See reference 6. We note that a strong attraction is automatically consistent with the tentative conclusion of reference 3. It is not clear whether the experimental data of reference 3 are inconsistent with the other possibility of a repulsive, but strongly absorbing, interaction.

[10]R. H. Dalitz (private communication), and M. Ross and G. Shaw (private communication).

[11]P. T. Matthews and A. Salam, Phys. Rev. 110, 569 (1958); C. Goebel, Phys. Rev. 110, 572 (1958); K. Igi, Progr. Theoret. Phys. (Kyoto) 19, 238 (1958).

[12]S. F. Tuan, Phys. Rev. (to be published); also University of California Radiation Laboratory Report UCRL-8461 (unpublished).

UPPER LIMIT FOR THE DECAY MODE $\mu \to e + \gamma$

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In theories of the Fermi interaction, it has been suggested that the weak interactions all arise from the coupling of a vector current with a heavy charged boson.[1] One of the consequences of the existence of such an intermediate boson is the decay mode[2] $\mu \to e + \gamma$ which would be expected to occur with a branching ratio

$$R(\mu \to e + \gamma)/R(\mu \to e + \nu + \bar{\nu}) \sim 10^{-4}.$$

Experimentally an upper limit for this branching ratio has already been found[3-5] to be $\sim 10^{-5}$. Although the calculation based on the idea of the intermediate boson is ruled out by these measurements, the status of the theory of weak interactions demands a close examination of the possible decay modes of the muon. We have therefore continued the experiment previously reported[4] and improved upon the result.

The search for the $\mu \to e + \gamma$ decay mode was made by observing coincidences between a gamma detector and an electron detector placed on either side of a source of decaying positive muons. These were obtained by stopping the 60-Mev π^+ beam of the Nevis cyclotron in a lithium target. There, the π^+ mesons decayed into muons whose range was too short for them to escape from the target. The decay products were then counted in the two telescopes. The experimental arrangement is shown in Fig. 1. An event was defined by a fast coincidence between counts in the electron and gamma-ray telescopes with the requirement that none of the anticoincidence counters triggered at the same time. All counts were gated so they were recorded only if they occurred during a cyclotron beam burst.

In addition to the electronic selection of the events, an event was checked by photographing pulses displayed on two oscilloscopes. On one, a sweep speed of 20 millimicroseconds per centimeter permitted a rough check on the timing of one counter pulse from the electron telescope relative to two from the gamma-ray telescope. Displayed on a second oscilloscope, with a relatively slow sweep speed, were pulses from the remaining coincidence counters as well as $2'$ and $3'$ anticoincidence signals before they were pulse shaped. The signature of an event is shown in Fig. 2.

The efficiency of the electron telescope to detect 53-Mev electrons was estimated to be 0.75.

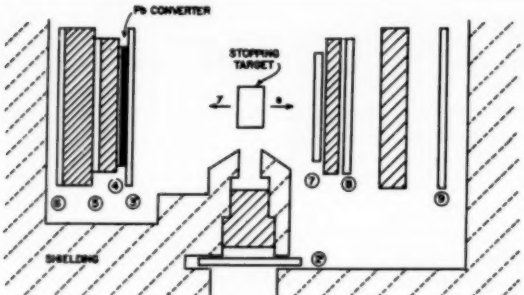

FIG. 1. The 60-Mev π^+ beam is stopped in the lithium target. Electrons are detected as a 789 coincidence. Photons are converted in the $\frac{1}{16}$-inch thick lead and are counted as a 456 coincidence in anticoincidence with 3′. All counters are plastic scintillators $\frac{1}{2}$ inch thick, except No. 4 counter which is $\frac{1}{4}$ inch thick. Counts coincident with incident beam particles were rejected by 2′. The shielding around the polyethylene absorber in the beam was lead and guarded against background from π^+ charge-exchange scattering.

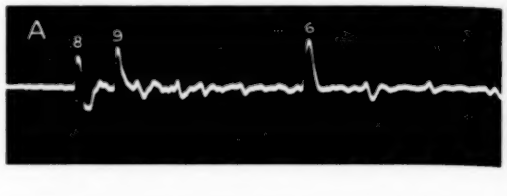

FIG. 2. Photographs of oscilloscope traces with a sweep speed of 1 μsec per centimeter. Trace A, with counter pulses 6, 8 and 9, is the signature of an event. One event was rejected on the basis of a trace which contained a pulse from the anticoincidence counter 3′. Trace B is an example of such an event.

The gamma-ray detector efficiency was calculated as a function of energy and was also experimentally determined with (a) the spectrum of gamma rays from the decays of π^0 mesons obtained by stopping the negative pion beam in a lithium hydride target,[6] and (b) attenuation measurements of electrons from the muon decays. The calculation and the measurements are in agreement, and from them is predicted a 0.15 probability for detecting 53-Mev gamma rays.

The sensitivity of the equipment to the $\mu \to e+\gamma$ decay mode was demonstrated by the observation of the radiative decay of the muon, $\mu \to e+\nu+\bar{\nu}+\gamma$, which, because of the simultaneous emission of an electron and gamma ray, is experimentally similar to the decay with no neutrinos. Several times throughout the run, the threshold energy requirements of the electron and gamma-ray detectors were lowered by removing the polyethylene absorbers, and under these conditions the measured counting rate was consistent with estimates of the radiative decay rate.[7] Other routine checks were made with the counters lined up in the direct meson beam, as well as by counting the gamma rays from the decays of charge exchange π^0 mesons produced by π^- captures in a lithium hydride target. For the latter test, a $\frac{1}{4}$-inch lead converter was placed in front of the electron telescope so that both detectors were sensitive to gamma rays.

The electron detector was used to measure the sample of decaying muons to which the detecting system was sensitive. Five counts were obtained as $e+\gamma$ coincidences out of 2.6×10^7 decay electrons. One of these counts was rejected upon examination of the slow oscilloscope photographs because an anticoincidence pulse failed to trigger electronically.

There are two sources of background which make significant contributions to the observed counting rate. One is the accidental counting rate between uncorrelated particles which register in both the electron and gamma-ray telescopes. The second is from real counts which originate from the radiative muon decay, $\mu \to e+\nu+\bar{\nu}+\gamma$. Backgrounds due to cosmic rays were eliminated by counting only during the cyclotron beam bursts, and by the use of an anticoincidence counter over the apparatus; while those which might arise from π^+ charge exchange scatterings were rejected by the 2′ anticoincidence counter placed in the pion beam.

The accidental rate was computed from the known gamma and electron counting rates and a measured value of the resolving time/cyclotron duty cycle. The last factor was obtained by measuring the accidental rate between uncorrelated decay electrons counted in both the electron and gamma-ray telescopes. For the sample of

muons observed, 1.5 accidental counts were expected. The contribution to the counting rate due to the radiative decay has been calculated,[7] but since there are no previous experimental data the result is not considered reliable to better than a factor of two. An absolute lower limit to the expected number of counts from these decays is then 1.4 counts.

On the basis of four events, the analysis of the data with the background as discussed above yields a branching ratio $R(\mu\to e+\gamma)/R(\mu\to e+\nu+\bar{\nu})$ which is probably less than 0.7×10^{-6}. The systematic error in the efficiency for detecting the $\mu\to e+\gamma$ decays is estimated to be not more than 30%, and the branching ratio is then found to be less than 2×10^{-6} with a 90% confidence level.

The authors are indebted to Dr. W. Chinowsky and Dr. A. Sirlin for discussions and to A. Garfinkel for his assistance with the experimental work, and they gratefully acknowledge the guidance and the valuable criticisms extended by Professor J. Steinberger throughout the course of the experiment.

[1]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

[2]G. Feinberg, Phys. Rev. 110, 1482 (1958).

[3]S. Lokanathan and J. Steinberger, Phys. Rev. 98, 240 (A) (1955).

[4]Berley, Lee, and Bardon, Post-Deadline Paper, American Physical Society Meeting, New York, January, 1959.

[5]Davis, Roberts, and Zipf, Phys. Rev. Lett. 5, 211 (1959).

[6]Lithium hydride was chosen over other hydrogenous materials (polystyrene, polyethylene, water), in which we also found a substantial number of π^- captures by hydrogen, because the low conversion probability in the lithium made this substance more favorable for efficiency measurements.

[7]The transition probability for the radiative decay was computed from the formulas derived by Behrends, Finkelstein, and Sirlin [Phys. Rev. 101, 866 (1956)], by assuming a $V-A$ interaction. An analysis of the data obtained is now in progress and a further communication on the subject of the muon decay with inner bremsstrahlung will follow.

CORRECTIONS TO THE $3D$-$2P$ TRANSITIONS IN μ-MESONIC PHOSPHORUS AND THE MASS OF THE MUON

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The recent precise determination of the muon magnetic moment[1] has raised the question as to whether the lower limit for the μ-meson mass can be trusted with enough precision to demonstrate a clear discrepancy between the experimental value for the muon moment and its value predicted by standard quantum electrodynamics. Actually, by using the best lower limit known up to now,[2] the inequality

$$g_\mu > 2(1.00158 \pm 0.00022), \tag{1}$$

obtains, whereas quantum electrodynamics predicts[3]

$$g_\mu = 2(1.001165). \tag{2}$$

The discrepancy of a few parts in 10^4 requires a precise knowledge of the precision to which the lower limit of the muon mass can be trusted. Crowe's best value for that limit involves a tentative limit of error of 2×10^{-4}, although some corrections were only guessed and some others were not yet evaluated. Owing to the real importance of having the most precise value of the limit, this paper is devoted to a very careful examination of the corrections in order to reduce considerably the limit of error. A summary of the corrections we have been considering is listed in Table I.

The fine structure splits the $3D$-$2P$ transition into a triplet, the components of which have the following energies in electron-volts:

$$3D_{3/2} - 2P_{3/2}\text{: } [1]\,(425.42)m_\mu/m_e; \tag{3}$$

$$3D_{5/2} - 2P_{3/2}\text{: } [9]\,(425.65)m_\mu/m_e; \tag{4}$$

$$3D_{3/2} - 2P_{1/2}\text{: } [5]\,(427.75)m_\mu/m_e. \tag{5}$$

The numbers in square brackets are the relative intensities.

Fitch, Rainwater, and Koslov[4] have shown that the $3D$-$2P$ transition energy falls above the Pb K-edge energy and below the Bi K-edge. Therefore the Pb K-edge energy will provide a lower limit to m_μ/m_e. Using the very probable value[5] of $(88\,015 \pm 5)$ ev for the Pb K-edge energy, one

gets, using (4), the inequality

$$m_\mu/m_e > 206.78 \pm 0.01. \quad (6)$$

Using (3), one would get a still higher lower limit. However, owing to its weakness, it is very likely that one cannot decide whether or not this energy is absorbed by the Pb filter.

The question is raised then whether one can without ambiguity decide, by analyzing the experimental curves,[6] if both the transitions $3D_{5/2}$-$2P_{3/2}$ and $3D_{3/2}$-$2P_{1/2}$ are absorbed by the Pb filter, or if only the $3D_{3/2}$-$2P_{1/2}$ energy is absorbed. In the latter case, the numerical value (6) would become an upper limit, whereas the energy of the $3D_{3/2}$-$2P_{1/2}$ transition would provide the lower limit:

$$m_\mu/m_e > 205.76 \pm 0.01. \quad (7)$$

The inequality (6), precise now to 5×10^{-5}, confirms the discrepancy between the experimental and theoretical values for the magnetic moment of the muon. However, if it would turn out that in fact the Pb edge splits the two transitions (4) and (5) of the triplet, the inequality (7) would no longer lead to a contradiction between theory and experiment[7]. But an improvement in this situation is also complicated by a thickness effect of the lead foil.[8] The authors do not feel competent to discuss this point any further.

In conclusion, if one accepts the results as stated above, the two possible explanations of the discrepancy between (1)[9] and (2) are as follows:

(a) Either the Pb K-edge actually splits the $3D_{5/2}$-$2P_{3/2}$ and $3D_{3/2}$-$2P_{1/2}$ lines of the $3D$-$2P$ transition in μ-mesonic phosphorus.

(b) Or the μ-meson behaves slightly differently from a simple heavy electron; the discrepancy would thus be the first sign of such a difference.

The authors are gratefully indebted to Professor L. Lederman for very instructive discussions on that matter.[10]

Table I. Corrections to the $3D_{5/2}$-$2P_{3/2}$ transition in μ-mesonic phosphorus ($Z=15$, $A=31$), not including reduced mass correction. ($m_\mu/m_e=206.75$ has been used.)

Effect	Contribution in ev (1 ry_∞ = 13.605 ev)
Second order vacuum polarization, corrected for reduced mass[a]	329.23
Fourth order vacuum polarization	2.19
Coulomb Green's function correction for vacuum polarization[b]	0.33
Finite-size effect on Coulomb potential[c]	-3.20±1.0
Shielding by atomic electrons	1.32
Lamb shift	-0.49
$(Z\alpha)^2$ correction to the Pauli approximation	0.16
Hyperfine structure (outer components: $f=2\rightarrow1$)	0.03
Finite-size correction to the vacuum polarization potential[d]	negligible
Finite-size corrections to the wave functions for vacuum polarization	...
Coulomb wave function corrections for second order vacuum polarization: not evaluated	±1.00
Corrections for finite mass of the nucleus (recoil)	negligible
(Notice that the $2P$ level has 4.3 ev natural width)	
Total	329.57±2.00

[a] A. Galanin and I. Pomeranchuk, Doklady Akad. Nauk S.S.S.R. 86, 251 (1952); L. Foldy and E. Eriksen, Phys. Rev. 95, 1048 (1954); A. Mickelwait and H. Corben, Phys. Rev. 96, 1145 (1954); S. Koslov, Nevis Report No. 19, 1956 (unpublished).

[b] E. Wichmann and N. Kroll, Phys. Rev. 96, 232 (1954).

[c] Using a spherical uniform charge density of radius $R=1.3A^{1/3}\times10^{-13}$ cm. We are grateful to R. Hofstadter for a discussion on this point.

[d] R. Rockmore (unpublished).

[1] Garwin, Hutchison, Penman, and Shapiro (to be published).

[2] K. Crowe, Nuovo cimento 5, 541 (1957); Cohen, Crowe, and DuMond, Fundamental Constants of Physics (Interscience Publishers, Inc., New York, 1957).

[3] C. Sommerfield, Phys. Rev. 107, 328 (1957); A. Petermann, Helv. Phys. Acta 30, 407 (1957); H. Suura and K. Wichmann, Phys. Rev. 105, 1930 (1957).

[4] Koslov, Fitch, and Rainwater, Phys. Rev. 95, 291 625 (1954).

[5] A. Sandström, Handbuch der Physik (Springer Verlag, Berlin, 1957), Vol. 30; Y. Cauchois and H. Hulubei, Tables de Constantes Sélectionnées. Longueurs d'Onde des Emissions X et des Discontinuités d'Absorption X. (Hermann et Cie, Paris, 1947). The uncertainty of 5 ev in the energy of the K-edge is taken for granted by the optimistic quotations of experts in this field. We thank Professor L. G. Parratt for private communication on the structure of the edge.

[6]S. Koslov, footnote a of Table I.

[7]G. Shapiro (private communication) has independently considered this possibility.

[8]J. W. M. DuMond (private communication). We are grateful to Professor DuMond for having called our attention to this very important feature.

[9]The uncertainty of 0.00022 being now reduced to 0.00005.

[10]G. Shapiro, at Columbia University also evaluated with D. Tycko the second-order vacuum polarization contribution on the IBM 650 computer, with the same result as we quote on the first line of the table.

THREE-BODY DECAYS OF K_2^0 AND K_1^{0*}

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In the course of our associated-production experiment using the Berkeley 10-inch liquid hydrogen bubble chamber, we have seen nine "anomalous" K^0 decays. Within their limited statistical accuracy these events (a) are consistent with equal leptonic decay rates for K_1^0 and K_2^0, (b) are in good agreement with decay rates predicted[1] by the "extended" $\Delta I = \frac{1}{2}$ rule, and (c) yield a new value for the K_2^0 lifetime.

In the entire experiment we find[2] 497 decays of the type $K_1^0 \to \pi^+ + \pi^-$ (that is, $N_{+-} = 497$), from K^0 produced via $\pi^- + p \to \Lambda + K^0$ or $\Sigma^0 + K^0$. The production and decay points are required to lie within a well-defined fiducial volume in the chamber. Of the nine K^0 decays which fail to fit $\pi^+\pi^-$ decay, one (previously reported[2]) fits $\pi^+\pi^-\pi^0$ decay ($N_\tau = 1$) and eight fit leptonic decay into $\pi^\pm\mu^\mp\nu$ and $\pi^\pm e^\mp\nu$ ($L = 8$). The incident π^- momentum is known precisely.[3] Therefore the K^0 momentum is known from its production angle. (There are actually four possibilities, corresponding to Λ and Σ^0 production, and to forward and backward c.m. production.) For given rest-mass assignments to the two charged decay fragments, and from their measured momenta, we can determine the missing energy and momentum, and therefore the rest mass of the neutral decay fragment. The errors are such that it is fairly easy to distinguish between the $\pi^+\pi^-\pi^0$ decays (135-Mev neutral rest mass) and the leptonic decays (zero neutral rest mass) and to eliminate all but one possible K^0 momentum. However, the four leptonic modes are not easily distinguishable among themselves, since the total energies of the charged decay fragments are determined largely by the momenta rather than by the rest masses. With a larger sample of data, a statistical separation would be possible.

A leptonic K^0 decay can escape detection by simulating a $\pi^+\pi^-$ decay. From the available phase space and known measurement errors, we estimate that less than 10% of the three-body decays are thus masked. No corresponding correction was made to L.

The events are listed in Table I and a photograph of one of the decays is shown in Fig. 1.

The "true" number of K^0 produced in the experiment is 2020 ± 100.[2] According to CPT invariance, half of these (K_1^0) should be short-lived ($N_1 = 1010$) and half (K_2^0) long-lived ($N_2 = 1010$).[4] Gell-Mann[5] has shown that if CP invariance holds, and if the weak interactions are not such as to allow $\Sigma^+ \to n + e^+ + \nu$, then K_1^0 and K_2^0 should undergo leptonic decay at the same rate,

$$\Gamma_{1L} = \Gamma_{2L}. \tag{1}$$

(The oscillatory interference terms between K_1^0 and K_2^0 disappear in the sum over both signs of electric charge of the decay products.)

There are two ways in which we can check the prediction (1). The first is to look at the time

Table I. K^0 three-body decays. T is the K^0 proper potential time and t the proper lifetime.

Event	P_K (Mev/c)	T (10^{-10} sec)	t (10^{-10} sec)
203 999	615	2.60	1.29
235 805	760	1.20	0.56
288 517	670	2.87	1.54
359 058	680	3.60	1.21
385 627	684	2.87	0.67
416 759	656	3.79	1.63
448 646[a]	298	4.89	2.32
499 237	240	9.16	3.81
501 242	120	13.22	0.20

[a]Decays into $\pi^+\pi^-\pi^0$. (The remaining eight decays are leptonic.)

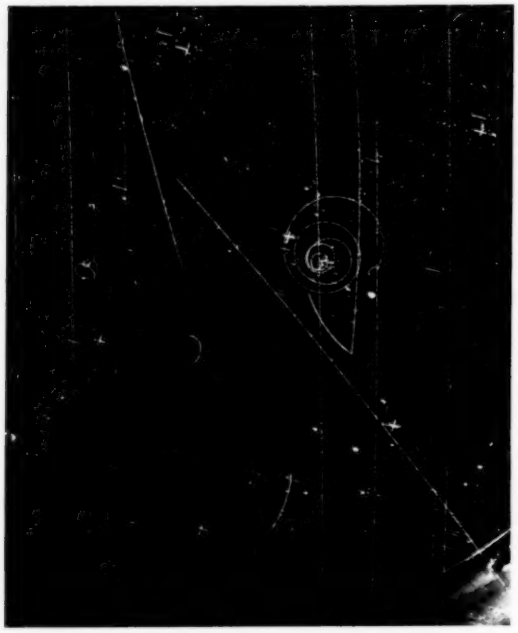

FIG. 1. Event 416 759. The production process is $\pi^- + p \rightarrow \Sigma^0 + K^0$, $(\Sigma^0 \rightarrow \Lambda + \gamma)$. The Λ decay into $p + \pi^-$ occurs closest to the production point. The other vee is best fitted by $K^0 \rightarrow \pi + e + \nu$. A large unbalance in the "visible" transverse momentum is obvious by inspection in the K^0 decay.

distribution of leptonic decays in the chamber. Decays from K_2^0 should be practically uniformly distributed over their potential proper times T. (T is the time interval in the K^0 rest frame between the K^0 production and the escape of the K^0—or of the center of mass of the decay fragments—across the boundary of the fiducial volume.) Therefore the number of leptonic decays from K_2^0 is given by

$$L_2 = N_2 \Gamma_{2L} \overline{T}, \tag{2}$$

where $\overline{T} = 3.21 \times 10^{-10}$ sec is the K^0 average potential time. Decays from K_1^0 should have, for a given T, the proper time distribution

$$dL_1 = N_1 \Gamma_{1L} \exp(-\lambda_1 t) dt, \tag{3}$$

between $t = 0$ and T. We attempt to distinguish L_1 from L_2 by constructing a likelihood function involving the flat distribution (2), and the exponential decay (3). The result is consistent (within one standard deviation) with either 100% K_1^0 or 100% K_2^0 decays. $\overline{T}$ is simply not long enough compared with the K_1^0 lifetime (for which we find $\lambda_1^{-1} = 0.94 \times 10^{-10}$ sec) to provide a sensitive test.

The second method of checking Eq. (1) makes use of the "Columbia" results for the K_2^0 lifetime and leptonic decay fraction measured by Bardon et al.[6] Their lifetime corresponds to the total decay rate

$$\Gamma_2(\mathrm{Col}) = \left(12.3^{+5.2}_{-3.5}\right) \times 10^6 \ \mathrm{sec}^{-1}. \tag{4}$$

They find no other K_2^0 decay modes besides $\pi\mu\nu$, $\pi e\nu$, and $\pi^+\pi^-\pi^0$, and find that 85 to 98% of the decays are into the leptonic modes. From Eq. (2) they can then predict the number of K_2^0 leptonic decays, L_2, expected in our experiment. By subtraction we can find L_1 and check Eq. (1). L_1 is obtained by normalizing to the $\pi^+\pi^-$ decays of K_1^0, since they have the same time distribution. Then

$$L_1 = N_{+-} (\Gamma_{1L} / \lambda_1 R_1), \tag{5}$$

where R_1 is the fraction 0.68 ± 0.04 of K_1^0 that decays into $\pi^+\pi^-$.[2] If we assume $\Gamma_{1L} = \Gamma_{2L}$, we can combine Eqs. (2) and (5) to obtain the total predicted leptonic decay rate,

$$L = \Gamma_{2L} [N_2 \overline{T} + (N_{+-} / \lambda_1 R_1)]. \tag{6}$$

In order to increase the sensitivity, we look only in the first K_1^0 mean life. Then the first term in (6) is reduced by a factor $\tau_1 / \overline{T}$ and the second by $1 - e^{-1}$. From the Columbia result Eq. (4), we then predict $L_1 = 0.5^{+0.2}_{-0.1}$ and $L_2 = 1.1^{+0.4}_{-0.3}$, or $L = 1.6^{+0.7}_{-0.4}$, which is to be compared to our three observed counts that occur between $t = 0$ and 0.94×10^{-10} sec. We thus find $\Gamma_{1L} / \Gamma_{2L} = 3.5^{+3.9}_{-2.7}$. Within the errors, Eq. (1) is satisfied. We will assume that Eq. (1) holds in what follows.

If one assigns isotopic spin $I = 0$ to leptons, then the hypothesis that there is a selection rule $|\Delta \vec{I}| = \frac{1}{2}$ can be "extended" to leptonic decays (e.g., $K^+ \rightarrow \mu^+ + \nu$ then satisfies the rule.) According to either the extended $|\Delta \vec{I}| = \frac{1}{2}$ rule or the "$I = \frac{1}{2}$ current" hypothesis[1,7] (which allows in general $\Delta I = \frac{3}{2}$ as well as $\frac{1}{2}$), the leptonic decay rates of K^+ and K_2^0 are related. One has $\Gamma(K_2^0 \rightarrow e^\pm \pi^\pm \nu) = 2\Gamma(K^+ \rightarrow e^+ \pi^0 \nu)$, and an exactly analogous relation with e replaced by μ. If we add these two relations, the left side becomes the total K_2^0 leptonic decay rate Γ_{2L}. The right side can be evaluated by using K^+ lifetimes[8] and branching ratios[9] as averaged by Gell-Mann and Rosenfeld.[5] The resulting predic-

tion[7] is

$$\Gamma_{2L} = (13.4 \pm 1.4) \times 10^6 \text{ sec}^{-1}. \quad (7)$$

Inserting our observation of $L = 8$ leptonic decays into Eq. (6) yields

$$\Gamma_{2L} = \left(20.4^{+7.2}_{-5.6}\right) \times 10^6 \text{ sec}^{-1}. \quad (8)$$

Our experimental result (8) is consistent with the prediction (7).

We now determine the K_2^0 lifetimes as follows. Corresponding to our one observed K_2^0 decay into $\pi^+\pi^-\pi^0$ ($N_\tau = 1$) there should be an additional 1.5 unobserved decays into $3\pi^0$.[10] (The decay of K_1^0 into $\pi^+\pi^-\pi^0$ should be negligible.[10]) Thus the K_2^0 decay rate into 3π is given by

$$\Gamma_{2\tau} = 2.5 N_\tau / N_2 \overline{T} = 7.7 \times 10^6 \text{ sec}^{-1}, \quad (9)$$

based on one event. Since there are no appreciable K_2^0 decay modes other than into 3π and leptons,[6] the total K_2^0 decay rate is given by adding our results (8) and (9) to obtain [subject to the assumption that Eq. (1) holds]

$$\Gamma_2(\text{UC}) = \left(28^{+10}_{-8}\right) \times 10^6 \text{ sec}^{-1}. \quad (10)$$

According to the $\Delta I = \frac{1}{2}$ rule (but not the $I = \frac{1}{2}$ current rule, except by accident) $\Gamma_{2\tau} = (6.0 \pm 0.4) \times 10^6$ sec^{-1} is predicted from the known K^+ decay rate into 3π.[1,10] After noting[2] the fortuitous agreement with our result (9), we combine this with the prediction (7) to yield a predicted[1] total K_2^0 decay rate

$$\Gamma_2(\Delta I = \tfrac{1}{2}) = (19.4 \pm 1.5) \times 10^6 \text{ sec}^{-1}, \quad (11)$$

in fair agreement with our experimental result (10).

Finally, since our K_2^0 decay rate (10) is in reasonable agreement with the result (4) of Bardon et al., we combine the two results to obtain[11]

$$\Gamma_2(\text{UC, Col}) = (16.3 \pm 3.5) \times 10^6 \text{ sec}^{-1},$$

in excellent agreement with the prediction (11) of the $\Delta I = \frac{1}{2}$ rule.

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*Work done under the auspices of the U. S. Atomic Energy Commission.

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[1]Okubo, Marshak, and Sudarshan, Phys. Rev. Lett. 2, 12 (1959).

[2]Crawford, Cresti, Douglass, Good, Kalbfleisch, Stevenson, and Ticho, Phys. Rev. Lett. 2, 266 (1959).

[3]Crawford, Cresti, Good, Stevenson, and Ticho, Phys. Rev. Lett. 2, 112 (1959).

[4]Lee, Yang, and Oehme, Phys. Rev. 106, 340 (1957), Eq. (31).

[5]M. Gell-Mann and A. H. Rosenfeld, in Annual Review of Nuclear Science (Annual Reviews, Inc., Stanford, 1957).

[6]Bardon, Lande, Lederman, and Chinowsky, Ann. Phys. 5, 156 (1958).

[7]Okubo, Marshak, Sudarshan, Teutsch, and Weinberg, Phys. Rev. 112, 665 (1958).

[8]Alvarez, Crawford, Good, and Stevenson, Phys. Rev. 101, 503 (1956); Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957 (Interscience Publishers, New York, 1957). V. Fitch and R. Motley, Phys. Rev. 101, 496 (1956); 105, 265 (1957).

[9]Alexander, Johnston, and O'Ceallaigh, Nuovo cimento 6, 478 (1957); Birge, Perkins, Peterson, Stork, and Whitehead, Nuovo cimento 4, 834 (1956).

[10]A. Pais and S. B. Treiman, Phys. Rev. 106, 1106 (1957).

[11]We combine the two lifetimes by constructing a likelihood function (LF) for each experiment and then multiplying them together to form a combined LF. The quoted errors correspond to a decrease of the combined LF by a factor exp(-1/2) from its maximum value. (This corresponds to 1 standard deviation for a Gaussian.) In terms of K_2^0 mean life the result is $\tau_2(\text{UC, Col}) = (6.1^{+1.1}_{-0.9}) \times 10^{-8}$ sec.

LOW-ENERGY APPLICATION OF RELATIVISTIC K-MESON–NUCLEON DISPERSION RELATIONS

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On the assumption that the K meson is pseudoscalar the approach of Chew et al.[1] has been extended to K-meson–nucleon scattering to find out whether the dispersion equations are compatible with the assumption of any resonance in the P states. The eight P-wave equations, under the assumed dominance of the P states in the dispersion integrals, can be written compactly as[2]

$$\mathrm{Re}h_\alpha(\omega)=\frac{\mu_\alpha}{N}+\frac{\lambda_\alpha}{\omega}+\frac{1}{\pi}\int d\omega'\left[\mathrm{Im}h_\alpha(\omega')\left\{\frac{1}{\omega'-\omega}+\frac{1}{N}\right\}+A_{\alpha\beta}\frac{\mathrm{Im}h_\beta{}^c(\omega')}{\omega'+\omega}\right], \tag{1a}$$

$$\mathrm{Re}h_\alpha{}^c(\omega)=\frac{\lambda_\alpha{}^c}{\omega}+\frac{1}{\pi}\int d\omega'\left[\mathrm{Im}h_\alpha{}^c(\omega')\left\{\frac{1}{\omega'-\omega}+\frac{1}{N}\right\}+A_{\alpha\beta}\frac{\mathrm{Im}h_\beta(\omega')}{\omega'+\omega}\right], \tag{1b}$$

where

$$\mu_\alpha=\begin{pmatrix}3f_\Sigma{}^2-f_\Lambda{}^2\\0\\f_\Sigma{}^2+f_\Lambda{}^2\\0\end{pmatrix},\quad \lambda_\alpha=-\mu_\alpha,\quad \lambda_\alpha{}^c=\tfrac{2}{3}\begin{pmatrix}f_\Lambda{}^2\\2f_\Lambda{}^2\\-f_\Sigma{}^2\\2f_\Sigma{}^2\end{pmatrix}, \tag{2a}$$

and

$$A_{\alpha\beta}=\tfrac{1}{6}\begin{pmatrix}1&-4&-3&12\\-2&-1&6&3\\-1&4&-1&4\\2&1&2&1\end{pmatrix} \tag{2b}$$

The 4×4 matrix $A_{\alpha\beta}$ is the present analog of the P-wave crossing matrix in the pion-nucleon case; it has the property $A^2=1$. The f_Y ($Y=\Lambda,\Sigma$) are the nonrationalized pseudovector coupling constants such that $f_Y{}^2=g_Y{}^2/4N^2$, where g_Y are nonrationalized but renormalized pseudoscalar coupling constants. The partial amplitudes h_α are given by

$$h_\alpha(\omega)=f_\alpha(\omega)/q^2\equiv e^{i\delta_\alpha(\omega)}\sin\delta_\alpha(\omega)/q^3,$$

where the phase shifts δ_α are real in the absence of inelastic processes. $\alpha=1,2,3,4$ refer to the 01, 03, 11, 13 states, respectively, the first suffix referring to the isobaric state and the other to the angular momentum one. q is the three-momentum of the meson in the barycentric system and ω is its total kinetic energy including the rest mass; the superscript c refers to the antiparticle, that is, to $\bar{K}$. Units have been so chosen that $\hbar=c=K=1$ and N is the nucleonic mass.

These P-wave equations, in the approximation in which one can ignore the absorption processes,[3] are of the form

$$\mathrm{Re}h_\alpha(\omega)=\frac{\lambda_\alpha}{\omega}+\int d\omega'\,F_\alpha(\sigma_\beta(\omega'),\omega',\omega) \tag{3a}$$

$$\simeq(\lambda_\alpha/\omega)+r_\alpha\lambda_\alpha+\cdots,\quad |\omega|<1. \tag{3b}$$

Comparing (3b) with (3a), one readily expresses the effective ranges r_α in terms of integrals over the various partial cross sections. The final expression (3b) can be approximately inverted to give the Chew-Low type equation

$$(\lambda_\alpha/\omega)\mathrm{Re}h_\alpha{}^{-1}(\omega)=(1-r_\alpha\omega). \tag{4}$$

For a resonance in an α state, it is clear that r_α must be positive; for only then does the cotangent of the (real) phase shift, δ_α, pass through zero. Conversely, if it is assumed that r_α is positive in any particular state, it is implied that the corresponding partial cross section should dominate in the various $F_\alpha(\sigma_\beta)$; this, in turn, should determine the signs of all the r_α, thereby checking the consistency of the original assumption.[4]

As an application of the above reasoning, we first consider the K particles. The most interesting point that emerges is that these relations do not produce any positive effective range.[5] The scattering lengths in $j=\frac{3}{2}$ states being zero, the question of resonance in those states does not arise; moreover, in order that r_3, say, may be positive one must assume that $\alpha=3$ state does not dominate and that either the 01 or 11 state of $\bar{K}$ particles should dominate. The first condition certainly contradicts the original assumption, while the second one is at variance with (1b) in the effective range approximation, as our subsequent discussion about $\bar{K}$ particles will show.[6] Thus the main and significant conclusion of our analysis of the K particles is that the dispersion relations give no grounds to expect any resonance

in the P states in spite of the similar space properties of π and K mesons.

For the $\bar{K}$ particles the situation is different since even at threshold the scattering is complicated by the production processes. But if one ignores them in the first approximation, one can get some insight into the behavior of these particles. An examination of the structure of the fourth row of (1b) immediately shows (in the effective-range approximation we are considering) that this state is certainly a candidate for resonance, for all the terms on the right-hand side are positive; that is, $r_4{}^C$ is positive definite. This is an exact analog of the 33 state in pion physics. But there is one remarkable difference in our case. The absence of a strong P state for the K particles leads to an over-all positive contribution by the integrals on the right-hand sides of (1b). Since $\lambda_2{}^C$ is the only other positive scattering length, the 03 state can equally claim to be a resonance state provided the coupling is strong. In other words, if one assumes that both the isobaric spin states in $j=3/2$ predominate, then we arrive at the conclusion that the dispersion relations are consistent with such assumption in the above sense.

We have further examined how our discussion of S waves is affected by the preceding analysis. On the basis of qualitative arguments it is concluded that the K-N scattering should be dominated by the S waves over a considerable range, and that the sign of the phase shifts at threshold should be given correctly by the Born approximation (corresponding to a repulsive interaction). For the $\bar{K}$ particles, in the absence of any experimental data in the moderately high-energy region, no definitive conclusion can really be drawn about their S-wave scattering. The results about the K particles are in agreement with experiment,[7] and lend further support to the assumption with regard to the K-meson parity.

The author would like to express his best thanks to Professor A. Salam and Dr. P. T. Matthews for suggesting the problem and for many instructive discussions.

[1]Chew, Low, Goldberger, and Nambu, Phys. Rev. 106, 1337 (1957).

[2]Details of the present work will be submitted for publication elsewhere.

[3]That the unphysical continuum contribution may be taken to be negligible has been effectively argued by P. T. Matthews and A. Salam, Phys. Rev. 110, 569 (1958).

[4]On the basis of this type of argument one can single out, in pion-nucleon case, the 33 state as the only consistent resonance state.

[5]Provided $g_\Lambda{}^2$ is not greater than $3g_\Sigma{}^2$; otherwise, the 01 state could be a resonance state but this would be reflected only in the total K-N scattering.

[6]Similarly one can show that r_1 is negative.

[7]Reports by M. F. Kaplon and R. H. Dalitz, in 1958 Annual International Conference on High-Energy Physics at CERN, edited by B. Ferretti (CERN, Geneva, 1958), pp. 171, 187; H. C. Burrowes et al., Phys. Rev. Lett. 2, 117 (1959).

EFFECT OF A PION-PION SCATTERING RESONANCE ON NUCLEON STRUCTURE*

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The electromagnetic properties of the nucleon have recently been studied by the dispersion-relation method.[1,2] Although qualitatively successful in accounting for the isotopic vector properties of the nucleon, these treatments proved incapable of explaining simultaneously the value of the magnetic moment and the radii of the charge and moment distributions.[3] The purpose of this Letter is to show that the inclusion of a strong pion-pion interaction could explain these aspects of nucleon structure.

Let us first consider the general formulation and solution of the dispersion relations. In the notation of reference 1, we write the following representations for the form factors[4]:

$$G_i{}^V(s) = \frac{1}{\pi}\int_{(2m_\pi)^2}^{\infty} \frac{g_i{}^V(s')\,ds'}{s'-s}, \tag{1}$$

$$G_i{}^S(s) = \frac{1}{\pi}\int_{(3m_\pi)^2}^{\infty} \frac{g_i{}^S(s')\,ds'}{s'-s}, \quad i=1 \text{ or } 2, \tag{2}$$

where $s=(p'-p)^2=(p_0'-p_0)^2-(\vec{p}'-\vec{p})^2$, the square of the energy-momentum transfer four-vector. The weight functions $g_i(s)$ are related to a sum over all virtual intermediate states that can be reached from a photon and lead to a nucleon-

antinucleon pair. For the isotopic vector functions $g_i^V(s)$, invariance considerations show that the least massive state is the two-pion state. We shall assume, as in previous treatments,[1,2] that the two-pion contribution dominates in the dispersion integrals. On the other hand, the least massive state contributing to $g_i^S(s)$ is the three-pion state. We have nothing to say here about this contribution and shall limit ourselves to the isotopic vector properties.

In the approximation stated above, $g_1^V(s)$ and $g_2^V(s)$ are proportional to the pion form factor multiplied by the appropriate projection of the amplitude for the process $\langle N\bar{N}|\pi\pi\rangle$ in the state $J=1$, $I=1$. Using the Mandelstam representation,[5] we are able to study the analytic properties of these projections, which we label $J_i(s)$. It can be shown that in the complex s-plane these functions are analytic except for branch cuts on the real axis for $s \leq 4m_\pi^2[1-(m_\pi^2/4M^2)]$ and $s \geq (2m_\pi)^2$. The right-hand branch cut, which was not considered in previous treatments, corresponds to π-π scattering. We shall show that it has an important effect on nucleon structure.

We can then write the dispersion relation

$$J_i(s) = \frac{1}{\pi}\int_{-\infty}^{a} \frac{\mathrm{Im}J_i(s')\,ds'}{s'-s-i\epsilon} + \frac{1}{\pi}\int_{(2m_\pi)^2}^{\infty} \frac{\mathrm{Im}J_i(s')\,ds'}{s'-s-i\epsilon}, \qquad (3)$$

where $a \equiv 4m_\pi^2(1-m_\pi^2/4M^2)$. Application of the unitarity condition shows that in the region $(2m_\pi)^2 \leq s \leq (4m_\pi)^2$, the phase of $J_i(s)$ is equal to the π-π scattering phase shift δ in the $J=1$, $I=1$ state. Considering only the two-pion intermediate state, we shall use this phase relation over the entire range of the right-hand integral in Eq. (3).

Because the left-hand integral is related to the pion-nucleon scattering amplitude, we shall consider it to be a known function. Equation (3) is then an integral equation, whose general solution has been found by Omnes.[6] In our case, his solution can be modified into the more tractable form

$$J_i(s) = e^{u(s)}\frac{1}{\pi}\int_{-\infty}^{a} \frac{ds'\ \mathrm{Im}J_i(s')}{s'-s-i\epsilon}\, e^{-u(s')}, \qquad (4)$$

where

$$u(s) = \frac{1}{\pi}\int_{(2m_\pi)^2}^{\infty} \frac{\delta(s')\,ds'}{s'-s-i\epsilon}. \qquad (5)$$

It can easily be seen that Eq. (4) reproduces the content of the integral equation (3); namely, $J_i(s)$ has the proper singularities, has the phase of π-π scattering on the right-hand cut, and has the correct imaginary part on the left-hand cut. If the integral defining $u(s)$ fails to converge, one can use the subtracted form

$$u_0(s) = \frac{s}{\pi}\int_{(2m_\pi)^2}^{\infty} \frac{\delta(s')\,ds'}{s'(s'-s-i\epsilon)}. \qquad (6)$$

We now require an expression for the pion form factor, $F_\pi(s)$, which satisfies the dispersion relation:

$$F_\pi(s) = 1 + \frac{s}{\pi}\int_{(2m_\pi)^2}^{\infty} ds' \frac{\mathrm{Im}F_\pi(s')}{s'(s'-s-i\epsilon)}. \qquad (7)$$

Unitarity allows us to conclude that the phase of $F_\pi(s)$ is the π-π scattering phase shift in the $J=1$, $I=1$ state,[7] and again we shall use this condition over the entire range of integration. In this case Eq. (4) degenerates into the solution

$$F_\pi(s) = e^{u_0(s)}. \qquad (8)$$

Combining Eqs. (4) and (8), we find

$$g_i^V(s) = |F_\pi(s)|^2 \frac{1}{\pi}\int_{-\infty}^{a} \frac{ds'\ \mathrm{Im}J_i(s')}{(s'-s-i\epsilon)F_\pi(s)}. \qquad (9)$$

Equation (9) reveals the important fact that, because of the phase conditions imposed by unitarity, it is the absolute value of the pion electromagnetic form factor which appears in the weight functions $g_i^V(s)$. Thus the well-known condition that $g_i(s)$ be real is satisfied.

Using Eqs. (9) and (1), let us now investigate the effect of π-π scattering on the nucleon structure. It has been shown by Drell[3] that in order to obtain agreement with the nucleon magnetic moment and radii, an enhancement of $g_i^V(s)$ by a factor of the order of five is required for $s < M^2$. From Eq. (9) it is apparent that a suitable peak in the pion form factor would produce this enhancement. We shall now show that a π-π resonance would result in such a peak.

An investigation of π-π scattering now in pro-

gress by Chew and Mandelstam has shown that the singularities of the partial-wave amplitude in the $J=1$, $I=1$ state are confined to branch cuts along the real axis in the range $s \leq 0$, $4m_\pi^2 \leq s$.[3] In the physical region, the effect of the left-hand singularities can be estimated by replacing the branch cut by a pole of appropriate position and residue. This approximation seems reasonable because for nucleon-nucleon scattering it leads to well-known effective-range formulas. Making this approximation, one finds the following solution for the $J=1$ state, for $\nu > 0$:

$$f_{\pi\pi}(s) = \left(\frac{\nu+1}{\nu^3}\right)^{1/2} e^{i\delta} \sin\delta = \frac{\Gamma}{\nu_\gamma - \nu[1-\Gamma\alpha(\nu)] - i\Gamma[\nu^3/(\nu+1)]^{1/2}}, \quad (10)$$

where

$$\nu = \tfrac{1}{4}s - m_\pi^2,$$

$$\alpha(\nu) = (2/\pi)[\nu/(\nu+1)]^{1/2} \ln[\nu^{1/2} + (\nu+1)^{1/2}].$$

A suitably continued form holds for $\nu < 0$. The constants Γ and ν_γ are determined by the position and residue of the pole. By examination of the structure of the π-π equations, Chew and Mandelstam have found that the sign of the residue must be positive, corresponding to an attractive force and raising the possibility of a resonance. Further theoretical information about the equivalent pole must await numerical solution of the very complicated π-π equations. We shall now show, however, that if the constants ν_γ and Γ are properly chosen, agreement with the nucleon-structure data may be achieved.

A properly normalized solution for $F_\pi(s)$ is

$$F_\pi(s) = f_{\pi\pi}(s)(s+s_0)/[s_0 f_{\pi\pi}(0)], \quad (11)$$

where s_0 is the position of the equivalent pole. The justification of this solution is, again, that it has the correct singularities and phase. Equation (11) clearly shows that a resonance in $f_{\pi\pi}$ will be reflected in the form factor F_π. The ob-

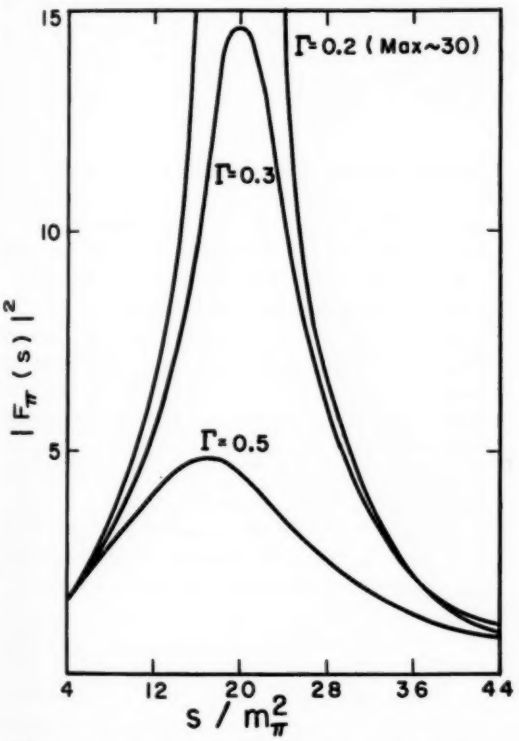

FIG. 1. The square of the magnitude of the pion form factor for $s \geq 4\mu^2$, for three values of the width Γ.

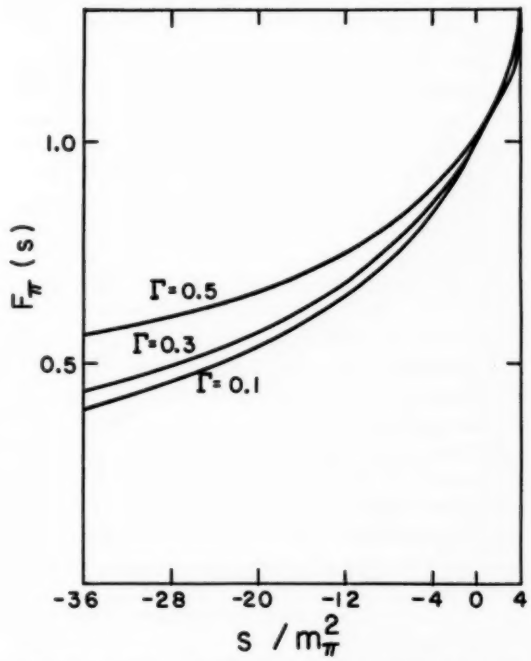

FIG. 2. The pion form factor in its physical region, for three values of the width Γ.

served values of the charge and magnetic-moment radii indicate that this resonance should occur at $\nu_\gamma \sim 3.5 m_\pi^2$ (square of the pion momentum in the π-π barycentric system). In Fig. 1 the function $|F_\pi(s)|^2$ is plotted for this value of ν_γ and several values of the width Γ. In Fig. 2 the pion form factor is plotted for $s < a$. Since it is less than one over most of this region, its appearance in the denominator of the integral of Eq. (9) will produce an additional enhancement.

In conclusion, our Eq. (9) for the weight functions together with the approximation given in Eq. (10) for the pion-pion scattering amplitude suggests that a π-π resonance of suitable position and width could lead to agreement between dispersion theory and many aspects of nucleon electromagnetic structure. Detailed calculations are in progress.

We are indebted to Professor Geoffrey F. Chew for his advice throughout this work, and for advance communication of some of the results on pion-pion scattering. We also acknowledge the help of James S. Ball and Peter Cziffra in obtaining Eq. (4).

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[1]Chew, Karplus, Gasiorowicz, and Zachariasen, Phys. Rev. 110, 265 (1958).

[2]Federbush, Goldberger, and Treiman, Phys. Rev. 112, 642 (1958).

[3]S. D. Drell, 1958 Annual International Conference on High-Energy Physics at CERN, edited by B. Ferretti (CERN, Geneva, 1958).

[4]If the integral in Eq. (1) fails to converge, one can use the usual subtracted form, as discussed in reference 1.

[5]S. Mandelstam, Phys. Rev. 112, 1344 (1958).

[6]R. Omnes, Nuovo cimento 8, 316 (1958).

[7]See, for example, Appendix II of Fubini, Nambu, and Wataghin, Phys. Rev. 111, 329 (1958).

[8]G. F. Chew, Lawrence Radiation Laboratory (private communication, 1959).

ENERGY OF THE GRAVITATIONAL FIELD*

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Einstein's equations for the gravitational field are valid for any system of coordinates and make it difficult for one to distinguish physical effects from effects of the curvature of the coordinate system. In consequence there is no obvious definition for the energy of the gravitational field. The usual definition, in terms of a component t_0^0 of the stress pseudotensor, makes the energy depend very much on the system of coordinates and is thus not satisfactory.

For physical problems one can restrict the gravitational field to be weak, of the order of γ, the gravitational constant. One can then use a system of coordinates for which the $g_{\mu\nu}$ differ from their values in special relativity by quantities of the order γ. Even if one restricts oneself to such coordinate systems (and renounces, for example, the use of polar coordinates), one can still make arbitrary changes of order γ in the coordinate system and such changes produce changes in the energy of the same order as the energy itself, so the difficulty persists.

In discussing the question people have usually lost sight of the primary requirement for the energy, that it shall be a useful integral of the equations of motion. The development of the Hamiltonian form of gravitational theory[1] enables one to make a new attack on the problem, taking this utility requirement into account.

In the Hamiltonian form one deals with the state at a certain time x^0, which state is described by dynamical variables for all values of x^1, x^2, x^3 for this one value of x^0. It is found that the only variables needed to describe the gravitational field are the six g_{rs} $(r, s = 1, 2, 3)$ and their conjugate momenta p^{rs}. The four $g_{\mu 0}$ do not enter into the description of the state at a certain time. They are needed only to provide a connection between the state at one time and the state at a neighboring time.

We are thus led to the condition that the energy at a certain time should involve only the g_{rs}, p^{rs}, and the nongravitational variables and should not depend on the $g_{\mu 0}$. The usual definition in terms of t_0^0 does not satisfy this condition, nor does the definition recently proposed by Møller.[2]

We shall take over the Hamiltonian worked out in reference 1, keeping the same notation except that we shall change the sign of all the $g_{\mu\nu}$ to make g_{00} negative (which entails changing also the sign of the $p^{\gamma s}$). Thus we have the Hamiltonian

$$H=\int\left\{(-g^{00})^{-\frac{1}{2}}\mathcal{H}_L+g_{\gamma 0}e^{\gamma s}\mathcal{H}_s\right\}d^3x, \qquad (1)$$

where $\mathcal{H}_L$ and $\mathcal{H}_S$ are functions only of the $g_{\gamma s}$, $p^{\gamma s}$, and nongravitational variables, namely

$$\mathcal{H}_L=K^{-1}(p^{\gamma s}p_{\gamma s}-\tfrac{1}{2}p_\gamma{}^\gamma p_s{}^s)+\tfrac{1}{4}K^{-1}(K^2e^{\gamma s})_v e^{uv}$$
$$\times(2g_{\gamma us}-g_{\gamma su})+\left\{K^{-1}(K^2e^{uv})_u\right\}_v+\mathcal{H}_{ML}, \qquad (2)$$

$$\mathcal{H}_S=p^{uv}g_{uvs}-2(p^{uv}g_{us})_v+\mathcal{H}_{Ms}, \qquad (3)$$

where a lower suffix added to a field quantity denotes an ordinary derivative. We have also the constraints

$$\mathcal{H}_L\approx 0, \quad \mathcal{H}_S\approx 0. \qquad (4)$$

In an ordinary Hamiltonian theory one takes the Hamiltonian itself to be the total energy, as it is always an integral of the motion. This will not do in the present theory, because, on account of the constraints (4), the Hamiltonian (1) equals zero. So long as one considers the exact equations of motion there does not seem to be any useful integral that one could take to be the total energy.

For the weak-field approximation it is reasonable to divide the Hamiltonian into two parts, a part that gives the main motion corresponding to $g_{\mu 0}=-\delta_{\mu 0}$ and a part that gives the correcting terms due to small deviation of the $g_{\mu 0}$ from the values $-\delta_{\mu 0}$. These two parts are

$$H_{\text{main}}=\int(w+\mathcal{H}_{ML})\,d^3x, \qquad (5)$$

where

$$w=p_{\gamma s}p_{\gamma s}-\tfrac{1}{2}p_{\gamma\gamma}p_{ss}+\tfrac{1}{4}(g_{\gamma su}g_{\gamma su}-g_{\gamma\gamma u}g_{ssu})$$
$$+\tfrac{1}{2}(g_{\gamma s\gamma}g_{uus}-g_{u\gamma s}g_{us\gamma}), \qquad (6)$$

and

$$H_{\text{cor}}=\int\left\{\tfrac{1}{2}(1+g_{00})(g_{\gamma s\gamma s}-g_{\gamma\gamma ss}-\mathcal{H}_{ML})\right.$$
$$\left.-g_{\gamma 0}(2p_{\gamma ss}-\mathcal{H}_{M\gamma})\right\}d^3x. \qquad (7)$$

The constraints in this approximation are

$$g_{\gamma s\gamma s}-g_{\gamma\gamma ss}-\mathcal{H}_{ML}\approx 0, \quad 2p_{\gamma ss}-\mathcal{H}_{M\gamma}\approx 0. \qquad (8)$$

The constraints now cause H_{cor} to have the value zero, but not H_{main}. The difference has arisen because of the neglect of a surface term at infinity in the derivation of (5), such neglect being justifiable because a term of this nature in the Hamiltonian does not influence the equations of motion.

Let us now consider an example for which, at large distances r from the origin, there is no matter present and $g_{\gamma su}$ and $p_{\gamma s}$ are of order r^{-2}. Such examples often occur in practice. We now have w of order r^{-4} at large distances, so H_{main} converges. It is a constant of the motion, provided we take values for the $g_{\mu 0}$ which preserve its convergence. H_{main} is now a useful integral of the motion, because its constancy is not a consequence merely of the constraints (8). We may thus reasonably define H_{main} to be the total energy.

For an example in which there is continuous emission of gravitational waves, $g_{\gamma su}$ and $p_{\gamma s}$ at great distances are of order r^{-1}. H_{main} now does not converge, corresponding to the total energy of the gravitational waves being infinite. The energy of physical importance is now the total energy within a large region R. To be able to obtain such an energy we need an expression for the energy density, at any rate for large values of r.

The expression (5) for the total energy in the case of convergence suggests that we look upon $w+\mathcal{H}_{ML}$ in general as the energy density, so that w is the energy density of the gravitational field. Let us examine whether this is permissible, taking first the special case when our coordinate system is such that

$$g_{\mu 0}=-\delta_{\mu 0}. \qquad (9)$$

The conservation of energy would require that

$$\partial(w+\mathcal{H}_{ML})/\partial x^0=k_{\gamma\gamma}, \qquad (10)$$

where k_γ is some 3-vector, which can be interpreted as the energy flux.

Now the first of the constraints (4), if evaluated from (2) to the second order in γ, gives

$$w+\mathcal{H}_{ML}=-\left\{K^{-1}(K^2e^{uv})_u\right\}_v. \qquad (11)$$

This leads immediately to (10) with

$$k_v=-\left\{K^{-1}(K^2e^{uv})_u\right\}_0.$$

But this k_v cannot be interpreted as the energy flux because it is of order γ, whereas the energy

density w in the absence of matter is of order γ^2.

We have, with the conditions (9),

$$\partial(w+\mathcal{H}_{ML})/\partial x^0 = \int [w+\mathcal{H}_{ML},\ w'+\mathcal{H}'_{ML}]d^3x'. \quad (12)$$

We see from (6) that w involves only undifferentiated momentum variables and dynamical coordinates differentiated not more than once, and we may assume that $\mathcal{H}_{ML}$ is similar. It follows that $[w+\mathcal{H}_{ML},\ w'+\mathcal{H}'_{ML}]$ cannot involve $\delta(x-x')$ differentiated more than once. Since it is antisymmetrical between the two points x and x', we can infer that it must be of the form

$$[w+\mathcal{H}_{ML},\ w'+\mathcal{H}'_{ML}] = (k_\gamma + k_{\gamma'})\delta_\gamma(x-x'), \quad (13)$$

for some 3-vector k_γ. Substituting this into (12), we get just the result (10). The gravitational part of k_γ, which comes from $[w, w']$, is linear homogeneous in the $p_{\gamma rs}$ and linear homogeneous in the $g_{\gamma rsu}$, so it is of order γ^2, the same as w. So this k_γ can be interpreted as the energy flux and the conservation law is verified.

When the conditions (9) do not hold, the above deduction gets spoilt by the extra change in $w+\mathcal{H}_{ML}$ produced by H_{cor}, given by (7). The source of the trouble is that $w+\mathcal{H}_{ML}$ gets altered if one makes a change in the coordinate system of order γ. The alteration consists of two parts, corresponding to the two terms in the integrand in (7). If the coordinates x^γ are changed by $x^\gamma \to x^\gamma + b^\gamma$, where the b^γ are functions of x^1, x^2, x^3 of order γ, the change in $w+\mathcal{H}_{ML}$ is

$$\delta_1(w+\mathcal{H}_{ML}) = -\int b_{\gamma'}[w+\mathcal{H}_{ML},\ 2p'_{\gamma ss'} - \mathcal{H}'_{M\gamma}]d^3x'. \quad (14)$$

If the hypersurface $x^0 = \text{constant}$ is changed by each point of it being shifted normally through a distance a, where a is a function of x^1, x^2, x^3, of order γ, the change in $w+\mathcal{H}_{ML}$ is

$$\delta_2(w+\mathcal{H}_{ML}) = \int a'[w+\mathcal{H}_{ML},\ g'_{\gamma s\gamma' s'} - g'_{\gamma\gamma s' s'} - \mathcal{H}'_{ML}]d^3x'. \quad (15)$$

The energy density, defined as $w+\mathcal{H}_{ML}$, is subject to these two uncertainties.

It may easily be verified that, when the conditions (9) hold, the component $t_0{}^0$ of the stress pseudotensor is just equal to w, so the conservation law proved above is not a new result. When the conditions (9) do not hold, $t_0{}^0$ differs from w by terms involving derivatives of the $g_{\mu 0}$. If the energy density is defined in terms of $t_0{}^0$, it is subject to the above two uncertainties and a further uncertainty because of its dependence on the $g_{\mu 0}$. So the use of w instead of t_{00} makes some improvement.

When the total energy is convergent it can be expressed as a surface integral at infinity. With the present definition of energy this integral is, according to (11),

$$-\int K^{-1}(K^2 e^{uv})_u \, dS_v. \quad (16)$$

With the usual definition in terms of the pseudotensor, it is

$$-\int\left\{(-g^{00})^{-1/2}K^{-1}(K^2 e^{uv})_u - g_{\gamma 0}p^{\gamma v}\right\}dS_v. \quad (17)$$

The two expressions agree provided $g_{\mu 0} = -\delta_{\mu 0} + O(r^{-1})$ at large distances.

On account of the uncertainties (14), (15), there does not seem to be any general definition for the energy density independent of the coordinate system. However, there is an important special case when these uncertainties vanish, namely, when there is no matter present and the gravitational field, to the first order of accuracy, consists only of waves moving in one direction.

In the absence of matter, (14) and (15) give, with the help of (6),

$$\delta_1 w = \tfrac{1}{2}g_{\gamma ru}(b_{uss} - b_{sus}) + g_{rs\gamma}b_{uus} - g_{\gamma su}b_{urs}, \quad (18)$$

$$\delta_2 w = -2p_{\gamma s}a_{\gamma s}. \quad (19)$$

Let us suppose the gravitational field consists only of waves moving, say, in the direction x^3. Then the derivatives $g_{\gamma su}$ and $p_{\gamma su}$ will vanish unless $u=3$. If we now make a change in the coordinate system so as to preserve the condition that the gravitational field consists only of waves moving in the direction x^3, we can introduce only coordinate waves moving in the direction x^3. This requires that the derivatives $b_{\gamma u}$ and a_u shall also vanish unless $u=3$. We now get $\delta_1 w = 0$ and

$$\delta_2 w = -2p_{33}a_{33}.$$

The second of the constraints (8) now gives $p_{\gamma 3} = 0$, so $\delta_2 w$ also vanishes.

We can conclude that the energy density of gravitational waves moving in a single direction is well-defined, independent of the coordinate system. It is only the interference energy density of waves moving in different directions that

is subject to uncertainties.

Let us go back to the problem of determining the total energy within a large region R surrounding some accelerating masses that are continuously emitting gravitational waves. Let us take a solution of the field equations in terms of retarded potentials without any ingoing waves. Then for large values of r, the main part of the gravitational field, of order r^{-1}, consists of waves moving in only one direction at each point, namely radially outward. The energy density w at large distances r is now well-defined, independent of any transformation of coordinates that preserves the character of the solution of being expressible in terms of retarded potentials, and does not introduce any ingoing coordinate waves.

The total energy within the region R, defined by

$$\int_R (w+\mathcal{H}_{ML})\,d^3x, \qquad (20)$$

is now also well-defined, because any transformation of coordinates that affects only the central part of R will not change (20), on account of (11), while any permissible transformation of coordinates in the outer part does not affect w and so does not affect (20). Thus the uncertainties in the energy density defined by $w+\mathcal{H}_{ML}$ do not affect calculations of the emission of energy by gravitational waves.

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[1]P. A. M. Dirac, Proc. Roy. Soc. (London) A246, 333 (1958).

[2]C. Møller, Ann. Phys. 4, 347 (1958).

POSSIBLE DETERMINATION OF HYPERON PARITIES AND COUPLING STRENGTHS

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The notions of "global" or "universal" symmetries, wherein the Σ and Λ hyperons are treated as members of the same multiplet structure in certain or all of their strong interactions, have been widely discussed,[1-3] although a decisive experimental determination of the relative Σ-Λ parity is yet to be carried out. These detailed theories require this relative parity to be even, but the possibility that it is odd has also been mentioned.[4,5] Several rather difficult experiments have been suggested to determine the relative Σ-Λ parity.[6-8] Furthermore, analyses of the forward angle K-meson–proton dispersion relations have been performed in order to determine simultaneously the relative Σ-Λ parity and the relative K-Λ parity.[9,10] It was emphasized some time ago,[11] and again recently,[12] that there are likely to be serious ambiguities in this procedure. In particular, one must at present make the somewhat subjective and certainly unjustifiable assumption that, in the case of odd relative Σ-Λ parity, the renormalized pseudoscalar coupling of K mesons is not an order of magnitude greater than the renormalized scalar coupling. In this note we shall discuss another experiment for determining the relative Σ-Λ parity which may be feasible at this time.

Chew[13] has suggested a method of extracting information from differential cross sections by their extrapolation to nonphysical values of momentum transfer. Thus an examination of the 400-Mev neutron-proton angular distribution in the backward directions has yielded a new evaluation of the pion-nucleon coupling constant.[14] We should like to suggest an analogous procedure applied to the hyperon-nucleon data concerning such processes as:

(a) $\Sigma^- + p \to \Lambda + n$,

(b) $\Sigma^- + p \to \Sigma^0 + n$,

(c) $\Sigma^\pm + p \to \Sigma^\pm + p$,

(d) $\Xi^- + p \to \Lambda + \Lambda$,

(e) $\Xi^- + p \to \Xi^- + p$.

Since at this time only processes (a) and (b) have been abundantly seen, we confine ourselves to a discussion of what we may be able to learn

from them. Both (a) and (b) may proceed through a direct channel, in which each baryon maintains its strangeness quantum number, or through an exchange process in which an odd number of K mesons is interchanged. The scattering amplitude, extended to nonphysical values of $\cos\theta$, will be assumed to possess singularities at such points that the scattering may arise from the exchange of a single real (on the energy shell) meson. In the "more-than-forward" direction ($\cos\theta > 1$) this may be accomplished by the direct process involving the exchange of one pion, while for $\cos\theta < -1$ a K meson need be exchanged. For the extrapolation to be possible it is necessary that the incident energy is sufficiently great so that the pole lies close to the point $|\cos\theta| = 1$. The following table indicates the required laboratory kinetic energies for the poles to occur at $|\cos\theta| = 1.1$ and 1.25, where θ is measured in the center-of-mass system.

	$\Sigma^- + p \to \Lambda + n$		$\Sigma^- + p \to \Sigma^0 + n$	
	$\cos\theta > 0$	$\cos\theta < 0$	$\cos\theta > 0$	$\cos\theta < 0$
$\lvert\cos\theta\rvert = 1.10$	250 Mev	2 Bev	210 Mev	2.5 Bev
$\lvert\cos\theta\rvert = 1.25$	105 Mev	700 Mev	85 Mev	1 Bev

An examination of the poles arising from the strangeness-exchange processes would require rather large Σ^- kinetic energies. However, a possible practicable application of this method is to the pion poles of processes (a) and (b). It may be observed that, unlike the neutron-proton case treated by Chew and Cziffra,[14] these processes are inelastic and consequently do not display diffraction peaks at forward angles. Moreover, there is no competing Coulomb peak such as would complicate the analyses of processes (c) and (e). Since the extrapolation extends only over a region of small momentum transfer, we might expect the scattering in this region to be dominated by the one-pion exchange. The contribution of this term to the matrix element is[15]

$$\frac{g g_{\Sigma\Lambda(\Sigma)\pi}}{(q^2+\mu^2)} (\bar{u}_{\Lambda(\Sigma)} \gamma_5^{1/2 \pm 1/2} u_{\Sigma})(\bar{u}_n \gamma_5 u_p), \qquad (1)$$

with ± 1 depending upon whether the relative $\Sigma^- - \Lambda(\Sigma^0)$ parity is even or odd, respectively. The contribution of this term to the cross section will display a second-rank pole at $q^2 = -\mu^2$. We have

$$(q^2+\mu^2)^2 \sigma(q^2, E) \propto g^2 g^2_{\Sigma\Lambda(\Sigma)\pi} \times [(m_\Sigma \pm m_\Lambda)^2 + q^2][4m_N^2 + q^2) + f(q^2, E), \qquad (2)$$

where $f(-\mu^2, E) = 0$. The extrapolation of the experimental function on the left-hand side of Eq. (2) to the location of the pole ($q^2 = -\mu^2$) will yield a large positive number for the case of even relative $\Sigma^- - \Lambda$ or $\Sigma^- - \Sigma^0$ parity and a small negative number for the case of odd relative parity. The difference is sufficiently marked, it would seem, to permit a determination of the relative $\Sigma^- - \Lambda$ or $\Sigma^- - \Sigma^0$ parity with even a relatively crude knowledge of the forward ($\theta < 45°$) angular distribution at ~100 Mev. Of course, with more accurate differential cross sections at higher energies one may also evaluate $g_{\Sigma\Lambda\pi}$ and $g_{\Sigma\Sigma\pi}$. Measurements of the slope of the left-hand side of (2), combined with estimates of $g_{\Sigma\Lambda\pi}$ and $g_{\Sigma\Sigma\pi}$ provide the method with a self-contained check that the differential cross section in this region is indeed dominated by the contribution from single-pion exchange.

Although we have considered only the extrapolation of forward-angle data concerning processes (a) and (b), an analysis of which might be expected in the not too distant future, it is reasonable to anticipate an improvement of the state of the art that would permit the extrapolation of higher energy backward angular distributions. From these one could, in principle, determine $g_{\Sigma NK}$, $g_{\Lambda NK}$ and the relative $K^+ - \Sigma$ and $K^+ - \Lambda$ parities. Moreover, from (c) we may learn $g_{\Sigma\Sigma\pi^0}$ from the forward behavior of the differential cross section, and the relative $K^0 - \Sigma$ parity (as well as $g_{\Sigma NK^0}$) from the backward behavior. The possibility of obtaining information on the relative Ξ^--nucleon parity and on the coupling strengths of pions and K mesons to Ξ particles is implicit in the application of these arguments to reactions (d) and (e).

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[1]M. Gell-Mann, Phys. Rev. 106, 1296 (1957).
[2]J. Schwinger, Ann. Phys. 2, 407 (1957).
[3]A. Pais, Phys. Rev. 112, 624 (1958).
[4]S. Barshay, Phys. Rev. Lett. 1, 97 (1958).
[5]F. Gürsey, Phys. Rev. Lett. 1, 98 (1958).
[6]A. Pais and S. Treiman, Phys. Rev. 109, 1759 (1958).
[7]G. Feldman and T. Fulton, Nuclear Phys. 8, 106 (1958).
[8]S. Barshay, Phys. Rev. 113, 349 (1959).
[9]D. Amati, Phys. Rev. (to be published).

[10]E. Galzenati and B. Vitale, Phys. Rev. (to be published). One of us (S.B.) would like to thank Dr. Amati and Dr. Vitale for communication of their results.

[11]S. Barshay, Phys. Rev. Lett. 1, 277 (1958).

[12]S. F. Tuan, University of California Radiation Laboratory Report UCRL-8461 (unpublished).

[13]G. Chew, 1958 Annual International Conference on High-Energy Physics at CERN, edited by B. Ferretti (CERN, Geneva, 1958), p. 96.

[14]G. Chew, reference 13, p. 97.

[15]In Eqs. (1) and (2), q^2 is the square of the nucleons' four-momentum transfer, E is the incident Σ^- total energy, μ is the pion mass, m_B is the mass of the baryon specified by the subscript B, g is the renormalized pseudoscalar pion-nucleon coupling constant, and the various renormalized hyperon-meson coupling constants are denoted by g with appropriate subscripts. The u_B are Dirac spinors.

ABSTRACTS

In this section are printed the abstracts of Articles that have been forwarded to The American Institute of Physics for publication in THE PHYSICAL REVIEW. In quoting information obtained from this section before the appearance of the corresponding Article, reference should be made to "Physical Review (to be published)" rather than to this Journal.

INTERACTION OF CHARGED PARTICLES WITH A DEGENERATE FERMI-DIRAC ELECTRON GAS. R. H. Ritchie, Health Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received November 17, 1958).

Expressions for the self-energy and interaction probability of charged particles in a degenerate Fermi-Dirac electron gas are given, generalizing from the dielectric theory of Lindhard and Hubbard and treating interactions between electrons in the gas by first-order perturbation theory. Numerical results for the interaction probability in a particular case are presented. A more general Feynman diagrammatic analysis of interaction in the gas is carried out, yielding results in agreement with the more elementary approach.

DIFFUSION OF RUTHENIUM IN SINGLE CRYSTALS OF SILVER. C. B. Pierce and D. Lazarus, Department of Physics, University of Illinois, Urbana, Illinois (Received December 5, 1958).

The diffusion coefficient of ruthenium in monocrystalline silver has been measured over the temperature range 793°C to 945°C, using Ru^{103} and Ru^{106} tracers, and a sectioning technique. The results may be expressed by an Arrhenius relation as $D=(180\pm70)\exp[-(65\,800\pm1000)/RT]$ cm^2/sec. The present result is consistent with earlier theoretical models, unlike the results obtained by Mackliet for Fe in Cu.

EXCHANGE ENERGY OF AN ELECTRON GAS IN A STRONG MAGNETIC FIELD. G. Dresselhaus, Department of Physics, Cornell University, Ithaca, New York (Received December 1, 1958).

The magnetic field variation of the exchange energy of a free-electron gas is calculated by deriving the form of the "Fermi hole" around each electron. The wave functions which are used exactly diagonalize the kinetic energy. The resulting charge distribution is integrated to find the exchange energy. The exchange energy has the same periodicity in the reciprocal magnetic field which is displayed by the kinetic energy. Hence, it is concluded that the de Haas-van Alphen effect is unchanged by the field variation of the exchange energy except for a possible shift in the phase of the oscillations.

STARK EFFECT FOR CYCLOTRON RESONANCE IN DEGENERATE BANDS. J. C. Hensel and Martin Peter, Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey (Received November 24, 1958).

A calculation of the motion is given for an electron in a simple band subjected to perpendicular magnetic and electric fields. It is shown that the cyclotron resonance frequency is unaffected by the presence of the electric field. For degenerate bands, however, there is a Stark shift of the cyclotron transitions between the low-lying "quantum" states. Calculations using second-order perturbation theory indicate that fractional line shifts of $\Delta\nu/\nu_0\sim10\%$ may be obtained under reasonable experimental conditions. This effect may be useful in the study of the valence bands of germanium and silicon.

ORDINARY HALL EFFECT IN Fe_3O_4 AND $(NiO)_{0.75}(FeO)_{0.25}(Fe_2O_3)$ AT ROOM TEMPERATURE. Jerome M. Lavine,* Division of Applied Science, Harvard University, Cambridge, Massachusetts (Received November 24, 1958).

Room temperature Hall measurements on a synthetic crystal of Fe_3O_4 and on a synthetic single crystal of $(NiO)_{0.75}(FeO)_{0.25}(Fe_2O_3)$ are reported. The thermoelectric power of the sample of Fe_3O_4 was also measured and verified that the sign of the charge carrier was negative. The ordinary Hall measurement of Fe_3O_4 suggests that the number of conduction electrons at room temperature is of the order of $3\times10^{21}/cm^3$, in rough agreement with Verwey's model. The ordinary Hall coefficient of $(NiO)_{0.75}(FeO)_{0.25}(Fe_2O_3)$ also suggests a large carrier concentration, of the order of $5\times10^{20}/cm^3$ at room temperature. The Hall mobility of Fe_3O_4 is of the order of 0.5 cm^2/volt-sec and the Hall mobility of $(NiO)_{0.75}(FeO)_{0.25}(Fe_2O_3)$ is of the order of 0.05 cm^2/volt-sec. The

conductivities of $(NiO)_{0.75}(FeO)_{0.25}(Fe_2O_3)$ and of a synthetic single crystal of $(NiO)_{0.56}(ZnO)_{0.14}(FeO)_{0.30}(Fe_2O_3)$ have been measured and compared with the conductivity of Fe_3O_4 employing a simple model for the mobility and associating the activation energy obtained from resistivity data with the number of conduction electrons. The observed data are in good agreement with the simple model.

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THERMAL RESISTANCE DUE TO ISOTOPES AT HIGH TEMPERATURES. V. Ambegaokar,* Westinghouse Research Laboratories, Pittsburgh, Pennsylvania (Received November 24, 1958).

In pure dielectric crystals, the isotopic variations of atomic mass contribute a temperature-independent thermal resistance at high temperatures. This resistance has been calculated using a Debye model of the vibrational spectrum. Umklapp processes are treated as the dominant scattering mechanism. The distribution of the heat current over the vibrational modes is known for low frequencies. This distribution is considered to apply over the whole frequency spectrum. The calculated result is compared with the experiments of Geballe and Hull on isotopically pure germanium. A satisfactory agreement with experiment is obtained.

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ANTIFERROMAGNETIC RESONANCE IN MnF_2. Fred M. Johnson* and Arthur H. Nethercot, Jr.,† Columbia Radiation Laboratory, Columbia University, New York, New York (Received November 25, 1958).

Measurements of the antiferromagnetic resonance frequency, $\nu(T)$, were made on single-crystal slabs of MnF_2 in the frequency range 96 to 247.2 kMc/sec and at temperatures, T, ranging from 4.2°K to 64°K ($T_N = 67.7$°K). The results are in general agreement with the resonance relations derived for antiferromagnetic materials by Nagamiya, Keffer and Kittel, and others. At low temperatures, values of $\nu(T)/\nu(0)$ and $\nu(0)$ are consistent with spin-wave method calculations of the sublattice magnetization and of the value and temperature dependence of the anisotropy energy. This agreement indicates almost complete correlation of adjacent electron spins in the low-temperature range as predicted from spin-wave theory.

From the antiferromagnetic resonance measurement of $\nu(0) = 261.4 \pm 1.5$ kMc/sec and Oguchi's calculation of H_A, $z|J|$ is 3.94×10^{-15} erg, in agreement with other determinations. Antiferromagnetic resonance line widths were measured from 4.2°K to 64°K. The observed asymmetric line shapes are satisfactorily accounted for as arising from a mixing of absorption and dispersion and from reflections. A simplified linewidth theory is given which satisfactorily accounts for the observed line widths except at the lowest temperatures, where a residual width is found. Finally, antiferromagnetic resonance measurements on two crystals, which were grown under different conditions, indicate no difference in resonance frequency or line width.

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MICROWAVE FIELD DEPENDENCE OF DRIFT MOBILITY IN GERMANIUM. Karlheinz Seeger,* Electrical Engineering Research Laboratory, University of Illinois, Urbana, Illinois (Received November 13, 1958).

At lattice temperatures between 80 and 300°K electrons and holes in germanium have been heated by microwaves, with an extension to high microwave fields of the technique developed by Morgan. The microwave frequency was 34.67 kMc/sec. The effect of carrier densities between 10^{13} and 10^{16} cm^{-3} has been investigated. At large microwave fields E (up to about 10^4 volts/cm in amplitude) the mobility μ, relative to the mobility μ_0 at zero field, is proportional to $E^{-\gamma}$; the exponent γ is smaller for purer samples. These microwave measurements agree with previous dc field measurements. At small values of E, $(\mu_0 - \mu)/\mu_0 E^2 = \alpha$ is field independent. The variation of α with lattice temperature T has been measured. In the range of pure lattice scattering, $\alpha(T) \propto \mu_0^2 T^x$; $\exp x = (n/n_0)^{0.67}$, where $n_0 = 1.7 \times 10^{14}$ cm^{-3} for electrons and 1.0×10^{14} cm^{-3} for holes. This dependence on carrier density suggests that carrier-carrier interaction plays an important role in the carrier-

phonon scattering mechanism.

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SLOW CAPTURE OF HOLES AND ELECTRONS BY SURFACE STATES ON GERMANIUM AND SILICON AT LOW TEMPERATURES. S. Roy Morrison, Honeywell Research Center, Hopkins, Minnesota (Received April 22, 1957; revised manuscript received December 8, 1958).

Field-effect decay curves at temperatures the order of liquid nitrogen are presented for n- and p-type germanium and for p-type silicon. A strongly asymmetric decay is observed, depending upon whether the induced charge is positive or negative. The decay is strongly influenced by illumination, temperature, and magnitude and sign of the field. Injected minority carriers also affect the decay rate. It is suggested that the trapping centers involved are the fast states, or recombination centers at the surface. After a disturbance of the charge in these states, the approach to steady state is slow due to the limited supply of minority carriers arriving at the surface. The results observed are compared with a model which takes surface barrier effects into consideration and are found in reasonably quantitative agreement.

PROMPT NEUTRON EMISSION FROM SINGLE FISSION FRAGMENTS. Stanley L. Whetstone, Jr., Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received November 26, 1958).

With a Cf^{252} source placed at the edge of a large cadmium-loaded liquid scintillator it has been possible to obtain a measure of the number of prompt neutrons emitted from a single fission fragment of measured mass. The mass determination results from a concurrent time-of-flight measurement of the two fragment velocities. The 30-in. scintillator has a high detection efficiency, little dependent on the energy of the neutrons, and permits detection in a full hemisphere about the direction of one of the fragment flight paths. The average number of neutron counts per fission is found to increase with the mass number of the fragment approaching the neutron detector in much the same way in both the light- and heavy-fragment groups, with a sharp decrease occurring in passing from the light to the heavy group. A correction for the geometry of the neutron detection, assuming isotropic emission of the neutrons in the fragment frames, enhances slightly the saw-tooth dependence of $\bar{\nu}(A)$ and gives for the ratio of the average number of neutrons from the light fragment to that from the heavy fragment $\bar{\nu}_L/\bar{\nu}_H = 1.02 \pm 0.02$. One is led to believe that either the emission of the neutrons is far from isotropic or that the slightly lighter fragments possess considerably more excitation energy than the slightly heavier fragments when the mass division is nearly symmetric. Perhaps a new picture of the mass division is indicated.

POLARIZABILITY OF THE NEUTRON. R. M. Thaler, Los Alamos Scientific Laboratory, Los Alamos, New Mexico (Received December 8, 1958).

Experiments on scattering of low-energy neutrons by heavy elements may give information concerning the electric polarizability of the neutron. The relation of the electric polarizability to the low-energy neutron scattering data is developed. One pertinent experiment is discussed, and from this an upper bound on the polarizability is obtained. This upper bound to the polarizability α is an order of magnitude larger than the meson-theoretic estimate of α. If the value of α is as small as is predicted by meson theory, or by an analysis of the pion photoproduction data, then it is unlikely to be observed in neutron scattering experiments of the presently achievable accuracy.

ELECTRIC POLARIZABILITY OF THE NEUTRON. G. Breit and M. L. Rustgi, Yale University, New Haven, Connecticut (Received November 17, 1958).

In connection with a proposed explanation of the anisotropy observed in the scattering of neutrons from various elements at energies of a few hundred kev, the order of magnitude of the neutron polarizability is estimated making use of data on pion photoproduction. A polarizability α greater than $\sim 2\times10^{-42}$ cm^3 appears unlikely on this basis. There remains an unexplained factor of ~ 50 which has to be accounted for either in the polarizability or by providing another explanation of the neutron-scattering anisotropy. The possibility of explaining the anisotropy on the basis of ordi-

nary scattering theory does not appear to be excluded. The exact proportionality of the coefficient of $\cos\theta$ to the neutron momentum does call for an r^{-4} type of potential, but it is not clear whether the energy dependence of the coefficient is sufficiently well determined by the data and whether the compound-nucleus features of the interaction are capable of explaining the observations. Nevertheless a few less usual effects are estimated. These are the interaction of the neutron moment with the vacuum polarization charge and with the external electric field of the nucleus as well as its interaction with the electric charge density at the nuclear surface. The latter is hard to distinguish from other nuclear effects. The former two effects are small and do not resemble the observed effects.

ILLUSTRATION OF THE DEPENDENCE OF THE SIGN OF δ OF A MIXED GAMMA-RAY TRANSITION ON ITS ORDER IN A CASCADE. S. Ofer,* Brookhaven National Laboratory, Upton, New York (Received December 2, 1958).

An analysis of the angular correlation functions obtained for γ-ray cascades following the decay of Tb^{156} (5 days) illustrates that different signs of δ (defined as the ratio, in a mixed transition, of the reduced matrix element corresponding to multipole order $L+1$ to that corresponding to multipole order L) have to be used in a case where a γ ray with mixed multipolarities is the first component of a cascade and in a case where the same γ ray is the second component of a cascade investigated, if the formulas and tables of Biedenharn and Rose are used for the analysis of the angular correlations.

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ELASTIC SCATTERING OF 40-Mev PROTONS FROM ISOTOPES OF Fe, Ni, AND Cu. M. K. Brussel* and J. H. Williams,† University of Minnesota, Minneapolis, Minnesota (Received November 26, 1958).

39.8-Mev protons were scattered from thin targets of Fe^{54}, Fe^{56}, Ni^{58}, Ni^{60}, and Cu^{65}. Absolute differential cross sections obtained with a statistical accuracy of $\lesssim 3\%$ have been determined for the elastically scattered protons. The range of the angular distributions, 7.5° to 110°, encompassed three minima and three maxima in the measured cross sections. The energy resolution of the detection equipment, utilizing a NaI(Tl) crystal, was 1.2–2%. This enabled a separation to be made of elastic from nonelastic events. A detector telescope allowed angular resolutions of $\pm 1/8°$ to be used in determining the shape of the features in the angular distributions. The general variation of the cross sections with the nuclear mass is noted. In addition, the data indicated that the nucleon shell closing about nucleon number 28 introduces fine structure differences in the shape and magnitude of features of the scattering pattern.

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EXCITATION FUNCTION OF THE REACTION $Zn^{64}(n,p)Cu^{64}$ WITH NEUTRONS OF ENERGIES BETWEEN 2 AND 3.6 Mev. J. Rapaport and J. J. van Loef, Laboratorio de Fisica Nuclear de la Universidad de Chile, Santiago de Chile (Received November 17, 1958).

The relative cross section for $Zn^{64}(n,p)Cu^{64}$ has been measured by an activation method for neutron energies of 2.0 to 3.6 Mev. The absolute cross section of 75.4 ± 11 mb at 3.55-Mev neutron energy is obtained by comparing the Cu^{64} positron activity with the amount of Si^{31} formed in the $P^{31}(n,p)Si^{31}$ reaction, which has a previously known cross section of 96.2 ± 9.0 mb. The cross section rises monotonically from a value of 12 mb at 2.0 Mev. The experimental results are compared with the predictions of statistical theory; experimental level densities are used in the calculations, but even so the predicted yield is too small. An upper limit of 2 mb is assigned to the cross section for $Zn^{68}(n,\gamma)Zn^{69*}$ at 3 Mev.

REACTION MECHANISM IN INELASTIC PROTON SCATTERING FROM Mg, Cr, AND OTHER ELEMENTS FROM 3.5 TO 7 Mev. Frederick D. Seward,* University of Rochester, Rochester, New York (Received November 26, 1958).

Inelastic proton scattering to the first excited states of Mg^{24} and Cr^{52} and to single excited states of other elements has been studied. $d\sigma/d\omega$ (90°) was measured as a function of proton energy from 3.5 to 7 Mev. Angular distributions of ine-

lastic protons and p'-γ angular correlations were taken at several energies. Experimental measurements are compared with direct-interaction theory and with predictions of the statistical model. Data for $Cr^{52}(p,p')Cr^{52*}$ at 5.4 Mev are well fitted by the statistical model. The $Mg^{24}(p,p')Mg^{24*}$ reaction at 7 Mev can be interpreted as a direct interaction. The Mg reaction at 5.4 Mev and the Cr reaction at 7 Mev appear to have a bit of direct interaction in them. It is suggested that the amount of direct interaction in this (p,p') reaction depends on the nuclear barrier height. Statistical-model expressions for reactions studied are given in an appendix.

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RESONANCE SCATTERING OF GAMMA RAYS BY THE 4.46-Mev AND THE 5.03-Mev STATES OF B^{11}. L. Cohen, R. A. Tobin, and J. McElhinney, Nucleonics Division, United States Naval Research Laboratory, Washington, D. C. (Received November 26, 1958).

The continuous x-ray spectrum from the NRL betatron was used to excite and study the 4.46-Mev and the 5.03-Mev states in B^{11}. The self-indication technique that was employed yielded the following values for the resonance integrals: $\sigma_0\Gamma = (0.79\pm0.21)\times10^{-4}$ Mev-barn for the lower state and $(0.98\pm0.17)\times10^{-4}$ Mev-barn for the upper state.

STUDY OF (d,α) REACTIONS ON SOME LIGHT NUCLEI. G. E. Fischer and V. K. Fischer, Columbia University, New York, New York and Brookhaven National Laboratory, Upton, New York (Received November 26, 1958).

The angular distributions of the charged particles from the $N^{14}(d,d)N^{14}$, $N^{14}(d,\alpha_0)C^{12}$, $N^{14}(d,\alpha_1)C^{12*}$, and $N^{15}(d,\alpha_0)C^{13}$ reactions have been studied with a deuteron bombarding energy of 21 Mev. The charged-particle groups are identified and their energy is measured by a dE/dx vs E counter telescope. The $N(d,\alpha)C$ angular distributions and the $O^{16}(d,\alpha)N^{14}$ angular distribution measured by Freemantle, Gibson, Prowse, and Rotblat have been compared with theoretical curves calculated from a simplified direct-interaction model. The relative magnitudes of the experimentally determined cross sections have also been compared with theory. The results indicate that the $O^{16}(d,\alpha_0)$ reaction can be described by the compound-nucleus extreme, while the $N^{14}(d,\alpha_1)$ process appears to favor description by a direct-interaction model. The remaining (d,α) reactions are intermediate cases.

ASYMMETRY IN THE DECAY OF Σ HYPERONS. R. L. Cool,* Bruce Cork, James W. Cronin,† and William A. Wenzel, Radiation Laboratory, University of California, Berkeley, California (Received December 5, 1958).

A search has been made for an asymmetry in the decay of $\Sigma^{\pm}$ hyperons. The $\Sigma^{\pm}$ were produced in the reaction $\pi^{\pm}+p\rightarrow\Sigma^{\pm}+K^{+}$. Detection and identification of the K^{+} mesons by a counter technique selects the above reactions and establishes the plane of production. Additional counters, which detect the pions from the Σ-hyperon decay in coincidence with the K^{+} mesons, measure the symmetry with respect to the plane of production. The results yield the parameter $\alpha\bar{p}$, where α measures the strength of parity nonconservation and $\bar{p}$ is the average polarization of the Σ hyperon. The data give: for Σ^{-} produced by 1.0-Bev π^{-}, $\alpha^{-}\bar{p}=+0.02\pm0.05$; for Σ^{-} from 1.1-Bev π^{-}, $\alpha^{-}\bar{p}=-0.06\pm0.05$; for Σ^{+} from 1.0-Bev π^{+}, the decay mode $\Sigma^{+}\rightarrow\pi^{+}+n$ gives $\alpha^{+}\bar{p}=+0.02\pm0.07$ and the mode $\Sigma^{+}\rightarrow\pi^{0}+p$ gives $\alpha^{0}\bar{p}=+0.70\pm0.30$. The value of $\alpha^{0}\bar{p}$ for $\Sigma^{+}\rightarrow\pi^{0}+p$ strongly indicates that parity is not conserved. For the Σ^{+}, $|\bar{p}|\geq0.70\pm0.30$; thus we can conclude $|\alpha^{+}|\leq0.03\pm0.11$. The results therefore indicate that the asymmetry parameter α depends upon the isotopic spin states in the final pion-nucleon system. The dependence on the isotopic spin is compatible with the $\Delta I=\pm\frac{1}{2}$ rule.

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SEARCH FOR DIRAC MONOPOLES. Hugh Bradner and William M. Isbell, Radiation Laboratory, University of California, Berkeley, California (Received November 24, 1958).

Nuclear emulsions were used to establish the following upper limits for the production of Dirac monopoles by 6.3-Bev protons:

If poles of protonic mass are produced by direct

nucleon-nucleon collision, the production cross section is less than 2×10^{-35} cm^2 per nucleon.

If poles of protonic or lower mass are produced by direct or by secondary processes and are subsequently held in matter with a binding energy less than 3 ev, the production cross section is less than 1.5×10^{-37} cm^2 per nucleon.

If poles of protonic or lower mass are produced by direct or by secondary processes and are subsequently held in matter with a binding energy between 3 ev and 20 ev, the production cross section is less than 10^{-40} cm^2 per nucleon.

ABSORPTION OF μ^- MESONS IN C^{12}. H. V. Argo, F. B. Harrison, H. W. Kruse, and A. D. McGuire, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received November 25, 1958).

It is known that there is a strong similarity between the electron-nucleon and electron-muon weak interactions. This paper is a report on an experimental investigation of the third leg of the triangle, the muon-nucleon interaction. The absorption of negative cosmic-ray muons stopped in C^{12} was studied, and the probability per second of absorption resulting in the formation of B^{12} in the ground state was measured and found to be 9050 ± 950 sec^{-1}. This is compared to the known rate of β decay of B^{12} to the ground state of C^{12}, 33.2 ± 0.65 sec^{-1}. The ratio of the rates is 273 ± 29. In the allowed approximation, the nuclear matrix elements for the two processes are the same, and the ratio of the rates can be calculated in terms of the ratio of the coupling constants without assuming a nuclear model. The short wavelength of the neutrino emitted in μ absorption (13×10^{-13} cm) causes forbidden matrix elements to make an important contribution to the μ-absorption rate, so that the theoretical prediction is dependent on the nuclear model. Within the uncertainties of the calculation, the electron-nucleon and muon-nucleon axial vector coupling constants are the same.

ENERGY DISTRIBUTIONS OF Li^8 FRAGMENTS EMITTED FROM C, Al, Cu, Ag, Au, AND U BOMBARDED BY 2.2-Bev PROTONS. Seymour Katcoff,* Chemistry Department, Brookhaven National Laboratory, Upton, New York (Received November 26, 1958).

Targets of C, Al, Cu, Ag, Au, and U were irradiated with 2.2-Bev protons at the Brookhaven Cosmotron. The secondary fragments were collected in nuclear emulsions placed at various angles to the beam. From a study of the numbers and lengths of the "hammer tracks," the energy distributions of the ejected Li^8 fragments were derived for each target element at two or more angles. Analysis of the results, and comparison with evaporation calculations for Cu, Ag, and Au targets indicated the following: (1) In general, the observed spectra show considerably more high-energy Li^8 fragments than the calculated spectra. (2) The higher the fragment energy, the greater the tendency for emission in a forward direction. (3) From Ag, Au, and U targets, Li^8 may be ejected partially by an evaporation mechanism, but some other process must also play an important role. (4) For C, Al, and Cu targets, evaporation of Li^8 fragments from residual nuclei does not seem to be operating to any appreciable extent. (5) The Li^8 spectrum from Cu is surprising in that it lies higher in energy by several Mev than the Li^8 spectrum from Ag. (6) The spectrum of Li^8 from U is very similar to that from Au; there is no evidence for emission of Li^8 fragments from excited fission products. (7) The cross section is estimated to increase monotonically from roughly one millibarn for Al to roughly ten millibarns for U.

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STUDY OF θ_1^0 PRODUCTION AND DECAY BY OBSERVATION OF ITS NEUTRAL DECAY MODE. John E. Osher, Burton J. Moyer, and Sherwood I. Parker, Department of Physics and Radiation Laboratory, University of California, Berkeley, California (Received November 25, 1958).

The production of strange particles possessing neutral-pion decay products has been investigated in proton bombardment of Bevatron targets. A well-collimated telescope counted energetic gamma photons originating within a few centimeters of the target. The position-dependent counting rate was determined by moving the telescope and collimation on a track parallel to the beam direction, and the resulting data were then compared to curves calculated from certain assumed kinematical models consistent with associated production. The counting rate and its

variation with position were consistent with the identification of $\theta_1^0 - 2\pi^0$, which implies even spin and even parity. The data also allow some gamma-ray contributions from $\theta^+ \to \pi^+ + \pi^0$, $\Sigma^+ \to p + \pi^0$, and $\Lambda^0 \to n + \pi^0$. The ratio of intensities from regions "upstream" to those "downstream" from the target implies a strongly peaked angular distribution, forward and backward, in the reference frame of the colliding nucleons which produce the θ_1^0.

PHOTON AND ELECTRON POLARIZATION IN HIGH-ENERGY BREMSSTRAHLUNG AND PAIR PRODUCTION WITH SCREENING. Haakon Olsen, Fysisk Institutt, Norges Tekniske Högskole, Trondheim, Norway, and L. C. Maximon,* Fysisk Institutt, Norges Tekniske Högskole, Trondheim, Norway, and Department of Theoretical Physics, The University, Manchester, England (Received November 24, 1958).

The matrix element for bremsstrahlung and pair production is written in a particularly simple form which reduces to the well-known matrix element for nonrelativistic energies, in terms of a vector which is closely related to the nonrelativistic current density vector.

The cross section for high-energy bremsstrahlung and pair production involving arbitrarily polarized photons and electrons has been calculated. The Coulomb and screening effects are taken into account exactly. It is found that the screening and Coulomb corrections to the polarization-dependent part of the cross section are analytically of the same form and numerically of the same magnitude as the corrections to the polarization-independent part of the cross section found earlier. We also give the cross section for bremsstrahlung summed over spin and polarization directions and integrated over the direction of motion of the final particle, i.e., the angular distribution of the radiation, which has a very simple form.

Formulas are given for the linear and circular polarization of bremsstrahlung from arbitrarily polarized electrons and for the spin polarization of pairs from arbitrarily polarized photons. The circular polarization of bremsstrahlung from longitudinally polarized electrons is complete at the upper end of the spectrum and is much larger than from transversely polarized electrons throughout the spectrum. In the same way, circularly polarized photons produce longitudinally polarized electron-positron pairs, the probability of producing transversely polarized pair particles being, in general, much smaller. The faster one of the pair particles is always polarized in the same sense as the photon. On the other hand, the linear polarization of bremsstrahlung is zero at the upper end of the spectrum and increases with decreasing photon energy. It is independent of the spin of the initial electron if one sums over the spin of the final particle. The circular polarization of the emitted photons is to a high degree independent of screening and Coulomb corrections, at the high energies considered here. The linear polarization is, however, significantly dependent on these corrections. Similar conclusions hold for pair production from circularly and linearly polarized photons, respectively.

Finally, the electron spin–photon polarization correlation for bremsstrahlung and pair production is discussed. The depolarization, because of bremsstrahlung, of polarized electrons passing through matter is calculated and is appreciable over a radiation length. It is greater for a transversely polarized electron than for a longitudinally polarized electron.

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MODIFIED ANALYSIS OF NUCLEON-NUCLEON SCATTERING: THEORY AND p-p SCATTERING AT 310 Mev. Peter Cziffra, Malcolm H. MacGregor,* Michael J. Moravcsik,* and Henry P. Stapp, Radiation Laboratory, University of California, Berkeley, California (Received December 5, 1958).

A modified method of analyzing nucleon-nucleon scattering is discussed and applied to proton-proton scattering experiments at 310 Mev. The modified scheme is based on an explicit inclusion in all higher-angular-momentum states of the Born approximation to the one-pion exchange process. This procedure is suggested by Chew's conjecture that the singularities of the scattering amplitude in the $\cos\theta$ plane (θ being the scattering angle in the center-of-mass system) that are closest to the physical region are due to the one-pion exchange process and are given by the Born approximation. Or, alternatively, in terms of ranges, the one-pion exchange contribution has the longest range of the forces contributing to the nucleon-nucleon interaction and

hence should be primarily responsible for the contributions to the scattering amplitude in the high-angular-momentum states. Since the only parameter in the Born approximation is the pion-nucleon coupling constant, the modified scheme can also provide a determination of this coupling constant. The application of the modified scheme to p-p scattering at 310 Mev indicates that the first two of the five best solutions of the conventional phase-shift analysis are more satisfactory than the others for two reasons. Firstly, their goodness-of-fit parameters improve markedly when the higher-angular-momentum contributions are added, whereas those of the others remain essentially unchanged. Secondly, as a function of the coupling constant, the goodness-of-fit parameters of the first two solutions show minima close to the accepted value of the coupling constant.

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A Q-NUMBER MODIFICATION OF THE LORENTZ ROTATIONS. V. Rojansky, Space Technology Laboratories, Los Angeles, California (Received August 20, 1958; revised manuscript received December 24, 1958).

The term $\rho_3 mc^2$ in the Dirac equation is replaced by $(\rho_3 a + \pi_3\rho_3 b_1 + \pi_1 b_2)mc^2$, where a, b_1, and b_2 are nonvanishing constants, and the π's are anticommuting square roots of unity that commute with the ρ's and the σ's. The resulting irreducible eight-component wave equation is not invariant under Lorentz rotations, but is invariant under similar transformations, in which the angle is multiplied by π_3. The question is raised whether such rotations through hyperbolic q angles may not be the proper quantum modification of the classical Lorentz rotations.

FIXATION OF COORDINATES IN THE HAMILTONIAN THEORY OF GRAVITATION. P. A. M. Dirac, Institute for Advanced Study, Princeton, New Jersey (Received December 10, 1958).

The theory of gravitation is usually expressed in terms of an arbitrary system of coordinates. This results in the appearance of weak equations connecting the Hamiltonian dynamical variables that describe a state at a certain time, leading to supplementary conditions on the wave function after quantization. It is then difficult to specify the initial state in any practical problem.

To remove the difficulty one must eliminate the weak equations by fixing the coordinate system. The general procedure for this elimination is here described. A particular way of fixing the coordinate system is then proposed and its effect on the Poisson bracket relations is worked out.

ELEMENTARY PARTICLES IN A FINITE WORLD GEOMETRY. H. R. Coish, Mathematical Physics Department, University of Manitoba, Winnipeg, Canada (Received October 20, 1958).

To account for the elementary particles a new physical geometry may be needed. A previous suggestion that this geometry may be finite is followed up by determining the representations of the orthogonal (Lorentz-like) group. Because of the existence of a new type of orthogonality-preserving transformation, some of the representations are multiple-valued. A change of value is identified with a gauge transformation, and electric charge is recognized as a certain number determining the many-valuedness of the representation. This charge number reverses sign under space inversion. The charged pions and sigma particles are correlated with some of the new representations.

ERRATUM

$H^3(\alpha,\gamma)Li^7$ AND $He^3(\alpha,\gamma)Be^7$ REACTIONS. H. D. Holmgren and R. L. Johnston [Phys. Rev. Lett. 2, 275 (1959) - Abstract].

Both expressions for the cross sections should be followed by mb instead of μb.